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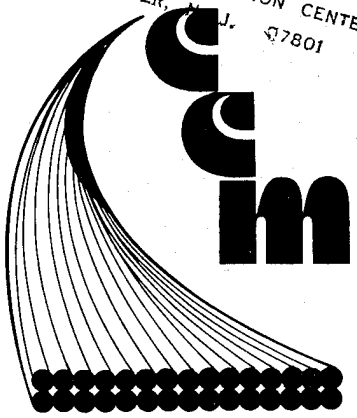
ANALYSIS OF THE "JOGGLE-LAP" JOINT FOR AUTOMOTIVE APPLICATIONS

RICHARD C. GIVLER



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Analysis of the "Joggle-Lap" Joint for Automotive Applications

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submitted as a Senior Thesis in partial fulfillment for a Degree with Distinction

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May 1, 1979

Abstract

An analytical model is developed to describe the response of the "joggle-lap" joint to both tensile and bending loads. The model consists of a non-linear beam analysis which calculates stress profiles through the adherent thickness. A plane stress finite-element model was incorporated into the analysis to correctly determine the stress field in the adhesive zone where it was shown that beam analysis was less accurate. Elastic response of the "joggle-lap" joint due to tensile loads was verified through experimental testing and ultimate loads were accurately predicted within experimental error. Maximum adherent flexural stress was found to determine joint failure. A parametric study was undertaken by using the verified analytical model and the results were recorded as a series of design curves.

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Nomenclature

a	cross-sectional area
b	adhesive bond thickness
C_0, C_1, C_2	constants
DEFLA	deflection
DUDSA	angular rotation
e	eccentricity
e_ℓ	elongation
E	modulus of elasticity for an isotropic material
E_f	modulus of elasticity of fibers
E_m	modulus of elasticity of matrix
E_n	modulus of elasticity normal to the fiber plane
E_x, E_y, E_z	modulus of elasticity of a general anisotropic body
F	applied force
$F_{i\ i=1-5}$	force components
$h_{i\ i=1-5}$	nodal points
$H_{i\ i=1-5}$	axial force of SEGi
I	moment of inertia about neutral surface
$I_{eq.}$	equivalent moment of inertia
ℓ_1	length of SEG1
$M_{i\ i=1-5}$	applied moment at SEGi
M_{corr}	correcting moment
$M(S)$	moment distribution

Nomenclature (Cont'd)

R	radius of curvature
$SEGi_{i=1-5}$	beam elements of the "joggle-lap" joint
S	specific gravity
$S_{i\ i=4-6}$	ultimate shear strength
t	adherent thickness
$\bar{u}$	distance between neutral axis and centroidal axis
u_o	deflection at the end of SEG1
$u_i, s_{i\ i=1-5}$	local coordinate system corresponding to individual beam element
v_f	volume fraction of fiber
v_m	volume fraction of matrix
$V_{i\ i=1-5}$	shear force on SEGi
W	weight fraction
X,Y	global coordinate system
Y	radial coordinate in curved beam members
X_i^T	ultimate tensile strength
X_i^C	ultimate compressive strength
α	angle measure
Δ	infinitesimal difference
$\varepsilon_x, \varepsilon_y, \varepsilon_z$	strain components
ε_{ult}	ultimate strain
Θ	angle measure
λ	linear measure

Nomenclature (Cont'd)

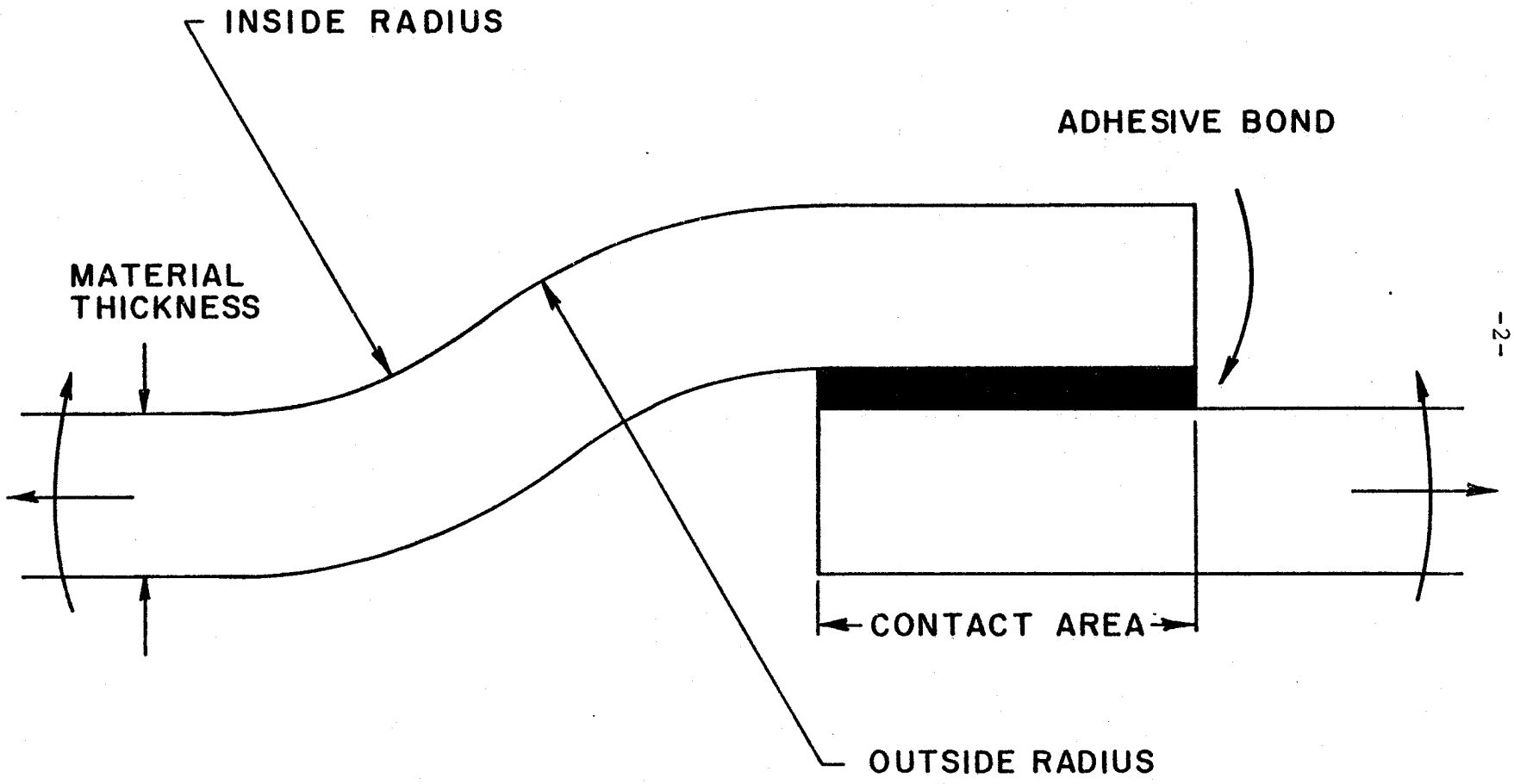
ν_{ij}	Poisson's ratio
π	3.14159...
$\sigma_1, \sigma_2, \tau_{12}$	plane stress components
$\sigma_x, \sigma_y, \sigma_{xy}$	
σ_{ult}	ultimate strength
ϕ	angle measure

I. Introduction

Recent government regulations for increased gasoline mileage requirements have induced automobile manufacturers to seek light weight replacement material systems for existing metal parts. Since the automotive industry is a high volume operation, sheet molding compound (SMC) parts offer a feasible answer to the problem. The SMC molding time of from 1 to 3 min/piece depending on the size and thickness of the part is compatible with automotive assembly line production.

International Harvester et al are currently employing SMC molded body components on their vehicles to replace former sheet metal parts. This new direction has brought with it several problems, one of which is the design of adhesive joints. The joint must accommodate high rate fabrication techniques and provide optimum strength and durability. In addition, the joint must satisfy certain cosmetic requirements such as adjacent flush edges. With these criteria in mind, the "joggle-lap" joint has been chosen for detailed study and analysis. This joint configuration is shown in Figure 1. Since a joint of this type experiences a variety of loading conditions in practice, it was decided to model the joint

FIGURE 1: THE JOGGLE LAP JOINT SUBJECT TO TENSILE AND BENDING LOADS



in pure tension and pure bending. By superposition, it is apparent that any combination of these two loading conditions may then be constructed.

This work focuses on the development of an analytical model to describe the behavior of the "joggle-lap" joint due to both tensile and bending loading conditions. The first section utilizes small deflection beam theory for both straight and curved beam elements to obtain a solution for the displacement and stress fields of the joint. Included in this analysis is the derivation of the governing differential equations for the deflection of the curved beam.

The second section utilizes a finite-element model to reveal localized stress concentrations in the adhesive zone. Boundary conditions for the finite element model are obtained from a transformation of stresses in the deformed geometry to equivalent stresses in the undeformed geometry. This transformation of stresses is performed via a computer routine for ease of calculation.

Finally, experimental verification of the analytical predictions is reported along with a description of testing procedures. The maximum flexural stress is shown to correlate strength data and failure analysis. Also, the microstructure of the joint was examined as a possible explanation of the failure mode.

II. Background

A. Adherent Materials

The adherents of the proposed "joggle-lap" joint were composed of a random-fiber composite known as SMC-25. SMC is defined as a sheet molding compound that contains reinforcements with an average fiber length of approximately 1 inch (2.54 cm) with random orientation in the plane. The number 25 indicates that the composite is 25 percent glass fibers by weight. The major constituents of SMC are E-glass fibers and a styrenated polyester resin in the form of a paste. It is quite common to use mineral fillers during the manufacture of the paste to facilitate flow when molding or to obtain certain characteristics from the molded part such as a high resistance to flame or increased stiffness. Another prime reason for using fillers is the fact that they are much cheaper than the polyester resin itself and thus reduce the cost of materials. At times, chemical additives may also be introduced into the paste to serve as catalysts during the molding cycle.

The process of SMC manufacturing is a highly innovative one which is completely automated. Figure 2 (taken from Owens/Corning Fiberglass SMC Review) depicts

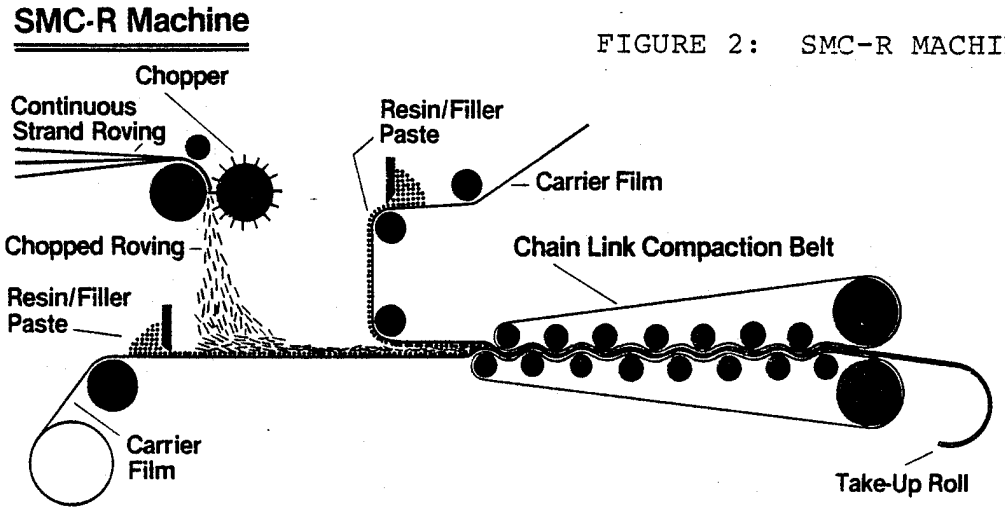


FIGURE 2: SMC-R MACHINE

a typical process currently in use by a competitive supplier of SMC. The first step of the procedure is to distribute the resin onto a polyethylene carrier film as shown. Continuous glass fibers are then chopped into lengths of less than three inches and distributed in a random fashion on the wetted film. A second layer of resin-coated polyethylene film serves as a top layer to the sandwich-like sheet. Several rows of rollers act to insure that the glass fibers are fully impregnated with the polyester resin thus yielding consistency in moldability of the SMC. Finally the product is directed to a take-up roll for ease of handling during shipping and storage.

SMC is usually placed in a constant temperature room while storing to allow maturation to take place. Maturation is nothing more than allowing the SMC to increase

in viscosity to enhance relative ease of handling of the sheet. Maturing the SMC sheet for extended periods of time greatly reduces the flow characteristics of the product while molding. Recommended shelf-life for SMC stored at 10-15^{o} C is about 2 weeks, however in general it may often be used up to 2 months after the date of its manufacture.

Once the SMC sheet has reached maturity, it is ready for molding. Upon removing the protective polyethylene film, the molding compound is cut to size and strategically placed in the mold. This procedure is known as charging the mold. The so-called strategic locations of the mold are those positions that allow the SMC to flow to all parts of the mold and maintain uniform part thickness. To date these locations have been determined by trial and error coupled with experience.

Compression molding combines both temperature and pressure to induce an exothermic reaction which serves to cure the part in the mold. Figure 3 (taken from ref [3]) is an example of a typical curing cycle showing the temperature of the part as a function of time. It should be noted that platen temperatures of 200^{o} C are usually sufficient for SMC molding and may be achieved with superheated steam. Another important fact seen from the figure is the overall cure time. Average cure times are generally

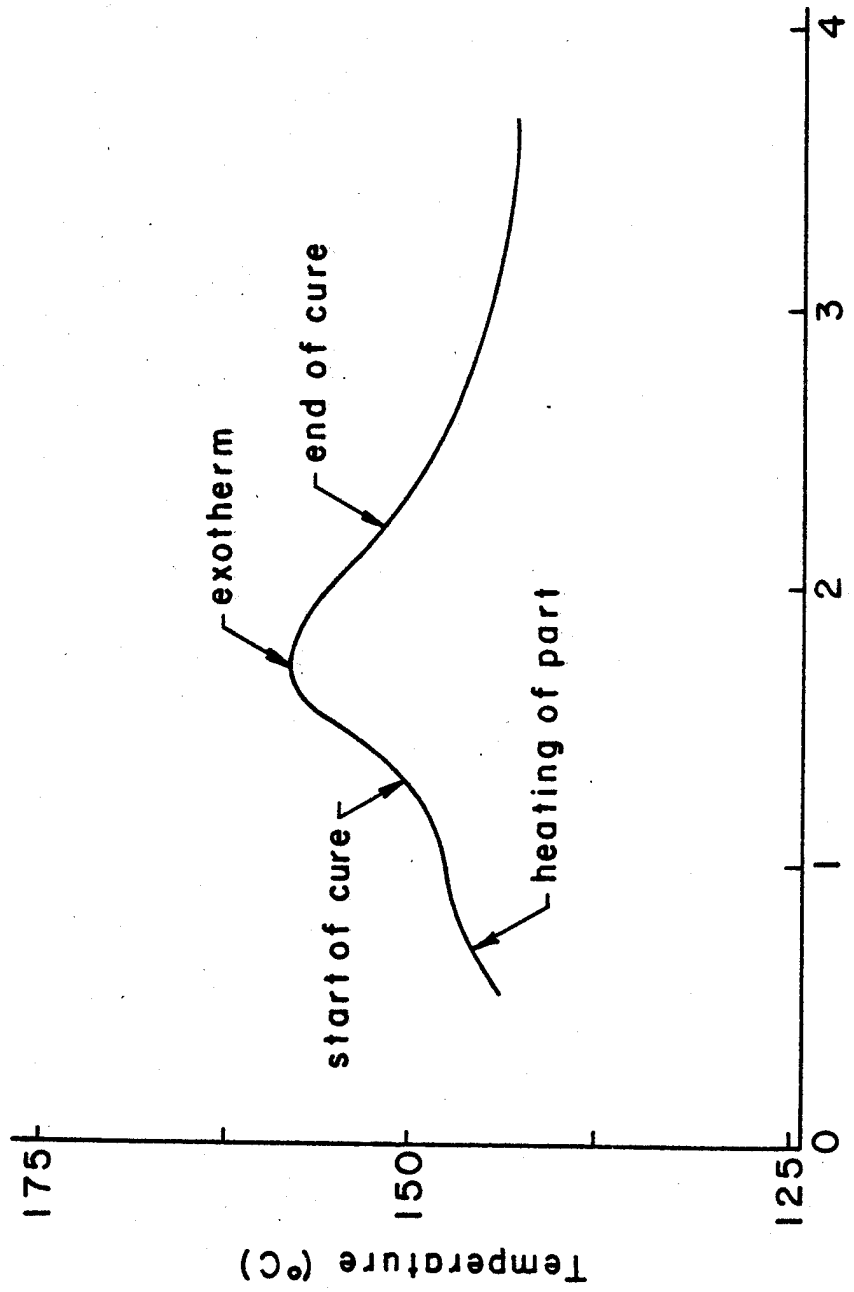


FIGURE 3: TYPICAL CURE CYCLE OF SMC

1-3 minutes (depending upon the thickness of the part) which lends itself to production line applications inherent in the automotive industry. Figure 4 (taken from ref [3]) shows the effect of pressure upon a typical cure cycle. Note that the peak pressure and maximum temperature correspond to the initiation of the exothermic reaction. The key to successful molding is to acquire fine control of the application of pressure to the cure cycle.

The main feature of SMC is the ability of the glass fibers to flow with the paste during the molding process. Since the fibers are transported to all parts of the mold, it is possible to produce a geometrically complicated part with quasi-constant mechanical properties. It has been shown by Pipes and Taggart [ref 5], that in areas of intensified flow, the fibers tend to align themselves with the direction of flow and thus produce areas of varying mechanical properties. It is therefore beneficial to understand the flow characteristics within the mold to produce a part with controlled and/or uniform mechanical properties. Taggart et al have determined the properties of SMC-25 to be those found in Table 1. Some scattering in the data was reported due to the inherent local variations in the material. To determine the normal modulus (modulus normal to the plane of the fibers), the relationship shown may be

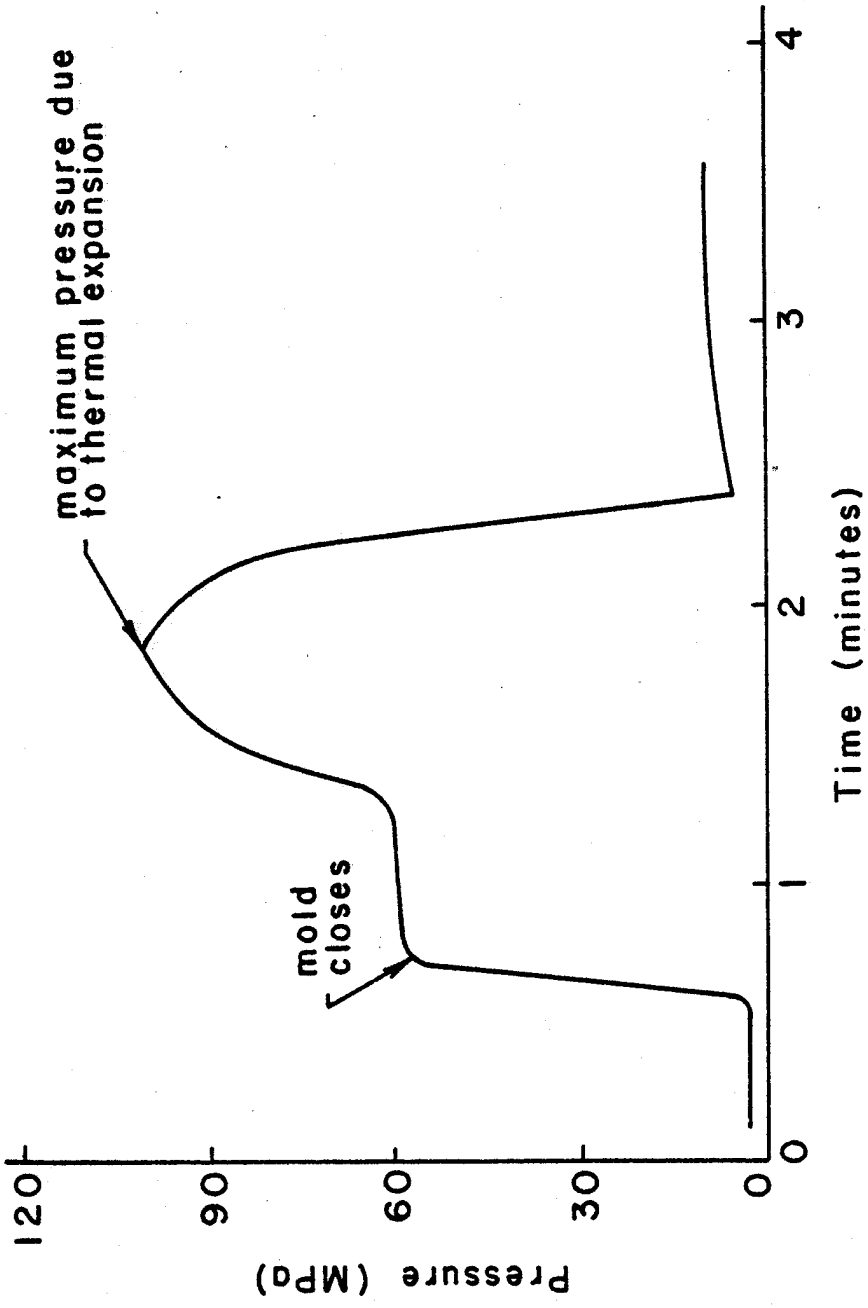


FIGURE 4: PRESSURE VARIATION OF A TYPICAL CURE

Table 1

Properties of SMC-25

Tension			
$E_{\perp}^{tension}$	GPa (Msi)	14.48	(2.1)
$\nu^{tension}$			.3
$\sigma_{ult}^{tension}$	MPa (ksi)	90	(13.1)
$\varepsilon_{ult}^{tension}$	(μ in/in)		11,400
Compression			
$E_{\perp}^{compression}$	GPa (Msi)	12.41	(1.8)
$\nu^{compression}$			.28
$\sigma_{ult}^{compression}$	MPa (ksi)	204	(29.6)
$\varepsilon_{ult}^{compression}$	(μ in/in)		20,600

used. This relationship resembles the well-known rule of mixtures for continuous fibrous composites.

$$\frac{1}{E_n} = \frac{v_f}{E_f} + \frac{v_m}{E_m} \qquad (1)$$

where E_n = normal modulus of elasticity of the composite

v_f = volume fraction of fiber

v_m = volume fraction of matrix

E_f = modulus of glass fiber

E_m = modulus of matrix

Table 2 provides the needed data for determining the normal modulus of elasticity. By definition, SMC is composed

Table 2[1]

	polyester resin	E-glass fiber
Modulus of Elasticity (10^6 psi)	.5	10
Specific gravity	1.28	2.54

of 25% fiber by weight. Utilizing the equation written below

$$v_f + v_m = 1 \qquad (2)$$

allows one to solve for v_f where v_m may be rewritten as

$$v_m = v_f \left[\frac{S_f}{S_m}\right] \cdot \left[\frac{W_m}{W_f}\right]$$

S_f = specific gravity of fiber

S_m = specific gravity of matrix

W_m = weight fraction of matrix

W_f = weight fraction of fiber

Making the appropriate substitutions, Eq. (2) becomes

$$\frac{.75}{.25}\left[\frac{2.54}{1.28}\right] v_f + v_f = 1$$

Thus the corresponding volume fraction of fiber and matrix are .14 and .86 respectively. From Eq. (1) the value of E_n is now calculated to be 0.58×10^6 psi.

---

[1] Vinson and Chou

B. Adhesive Materials

The adhesive system chosen for the "joggle-lap" joint was developed by the Adhesives Division of Goodyear Chemicals. The Pliogrip 6000 series is a general purpose structural adhesive with a polyurethane base. Currently available as a two-part system, Pliogrip 6000 exhibits both high flexibility and resilience. With the proper selection of curatives, the working time of the adhesive may be accurately controlled between 1-6 minutes.

In order to utilize this adhesive system only minimal surface preparation is necessary. The two surfaces to be bonded are prepared with a plastic wash primer (Pliogrip 6033/6034 Wash Primer) that is applied with a cloth. No sand blasting or surface stripping is necessary. To maintain reliably bonded parts, Pliogrip 6000 must be mixed at a precise ratio of 4 parts resin to 1 part curative by weight or volume. Deviations from this standard will yield resin-rich areas of uncured adhesive. The actual mixing of the two components must be carried out without the introduction of air into the system, thus the need for specialized equipment. Without this precaution, entrapped air bubbles in the cured adhesive would yield voids and greatly affect the performance of the bond. Curing this adhesive system can be accomplished at room temperature, however the use of heated fixtures will reduce cure times.

Recommended clamping pressures of heated fixtures range from 20 to 40 psi.

An important criterion in the design of bonded joints is that of the adhesive thickness. It has been shown that adhesive properties vary inversely with adhesive thickness. Thus the bulk properties of the adhesive are distinctly different from those in the film state. So the question is posed as to the optimum bond thickness as a function of shear strength. Figure 5 (taken from Pliogrip technical data, Goodyear Adhesives) shows the effect of glue line thickness on bond joint strength. A bond line thickness of 0.030 inches was chosen as optimal even though thicknesses less than 0.030 inches yield greater bond strengths. It was felt that bonding thicknesses less than 0.030 inches are not capable of being fabricated with consistency under production operations. (i.e. molded FRP parts will inherently not fit together with reliable precision).

To achieve uniform bond lines, one of two procedures is generally used. Adherents may have a small raised button of 0.030 inches in thickness which acts as a spacer for the joint to insure a uniform bond. Another procedure is to introduce small glass spheres (0.030 inches dia.) directly into the adhesive to achieve similar spacing. The effect of these spheres on joint strength has not been determined but it is argued that the variation from the norm is negligible.

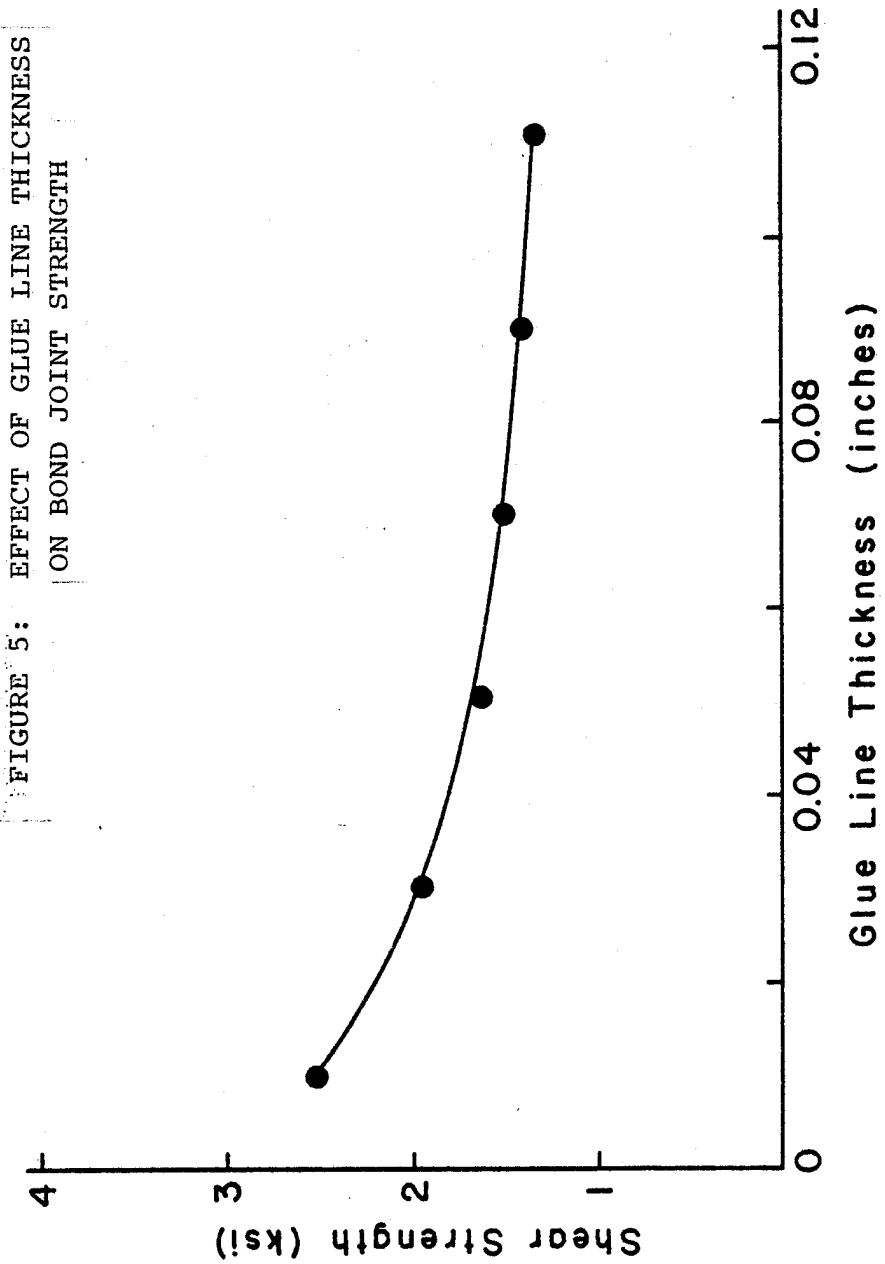


FIGURE 5: EFFECT OF GLUE LINE THICKNESS ON BOND JOINT STRENGTH

III. Methods of Analysis

A. Tensile Loading

a. Beam Model

Recently, Adkins [ref 2] investigated the response of a scarf joint to simple tensile loadings. It was found that the scarf joint exhibits flexural deformation under tensile loading due to the misalignment between the neutral surface and the loading axis. This eccentricity induces a moment distribution along the joint (see Figure 9) which acts to align the neutral surface with the loading axis.

The analysis of the "joggle-lap" joint, shown previously in Figure 1, is an extension of the concept discussed above. Again it is clear that under tension the joint will experience a lateral deflection as the neutral axis attempts to align with the applied force. To analyze the joint behavior under tensile loading conditions, it was decided to divide the joint into five segments. The obvious places to divide the joint are illustrated in Figure 6 along with the corresponding identifying labels and global coordinate system. Reference to beam segments via their identifying numbers will be utilized throughout the remainder of this analysis.

In general, the goal of the analysis will be to determine the displacements of the neutral axis as measured perpendicularly from the undeformed neutral surface. Once

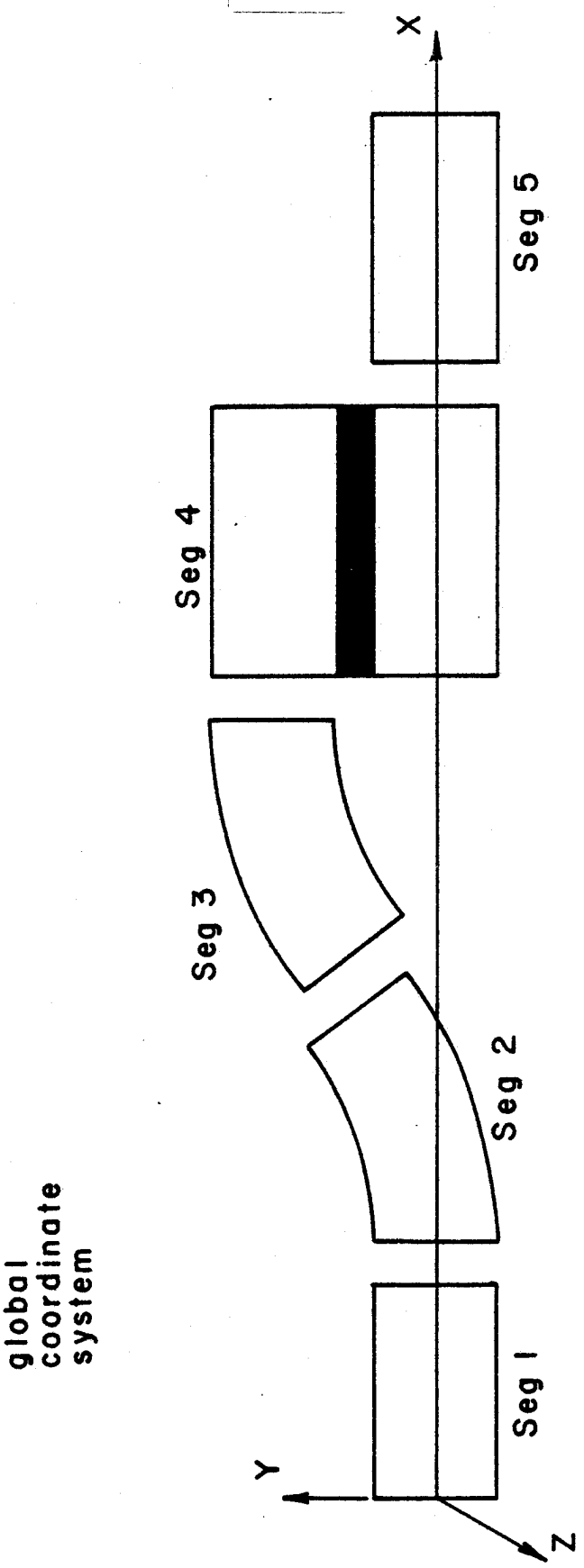


FIGURE 6: PIECEWISE REPRESENTATION
OF THE JOGGLE LAP JOINT

the deflections are known, one may calculate a moment distribution along the joint and thus determine the stress distribution at any given cross-section.

The initial intention of such an investigation was to develop a closed form solution for the stresses within the joint. This effort was soon thwarted by the non-linearities encountered in the governing equations for the beam elements. These non-linearities result from a coupling between the moment and deflection solutions, as will be evident later. As an alternative solution, the displacement field was obtained via numerical integration routines.

Linear elastic beam theory states that for a beam under general loading conditions, the local radius of curvature is given by

$$R = \frac{EI}{M} \tag{3}$$

where R = radius of curvature

E = modulus of elasticity

I = moment of inertia about the neutral surface

M = applied moment

The radius of curvature may be written in terms of the lateral deflection as given by Eq. (4)

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{[1 + (\frac{dy}{dx})^2]^{3/2}} \tag{4}$$

Realizing that under the assumptions made with regard to small deflection theory, the term $(\frac{dy}{dx})^2$ will be negligible when compared to unity. Thus one arrives at the governing equation for straight beam elements.

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \tag{5}$$

Since the material system is relatively stiff, it is assumed that small deflection beam theory will yield sufficiently accurate results. Thus, one may write a governing differential equation for each segment of the joint. By matching boundary conditions of deflection and slope at each interface, the deflection of the entire joint may be obtained as a function of distance along the neutral axis. Details of the analysis may be referenced in Appendix B.

To enhance one's understanding of the joint behavior under applied tensile loadings, Figures 7 through 10 show deflection, slope, moment, and shear diagrams respectively at a load of 200 lbs. Many of the discontinuities found in the plots arise from a shift in the neutral axis which is a common occurrence among lap joints.

It was stated previously that analyzing the "joggle-lap" joint under tension was a non-linear problem. This was seen by the fact that the moment was a function of the deflection. Another way to view the non-linearities of the

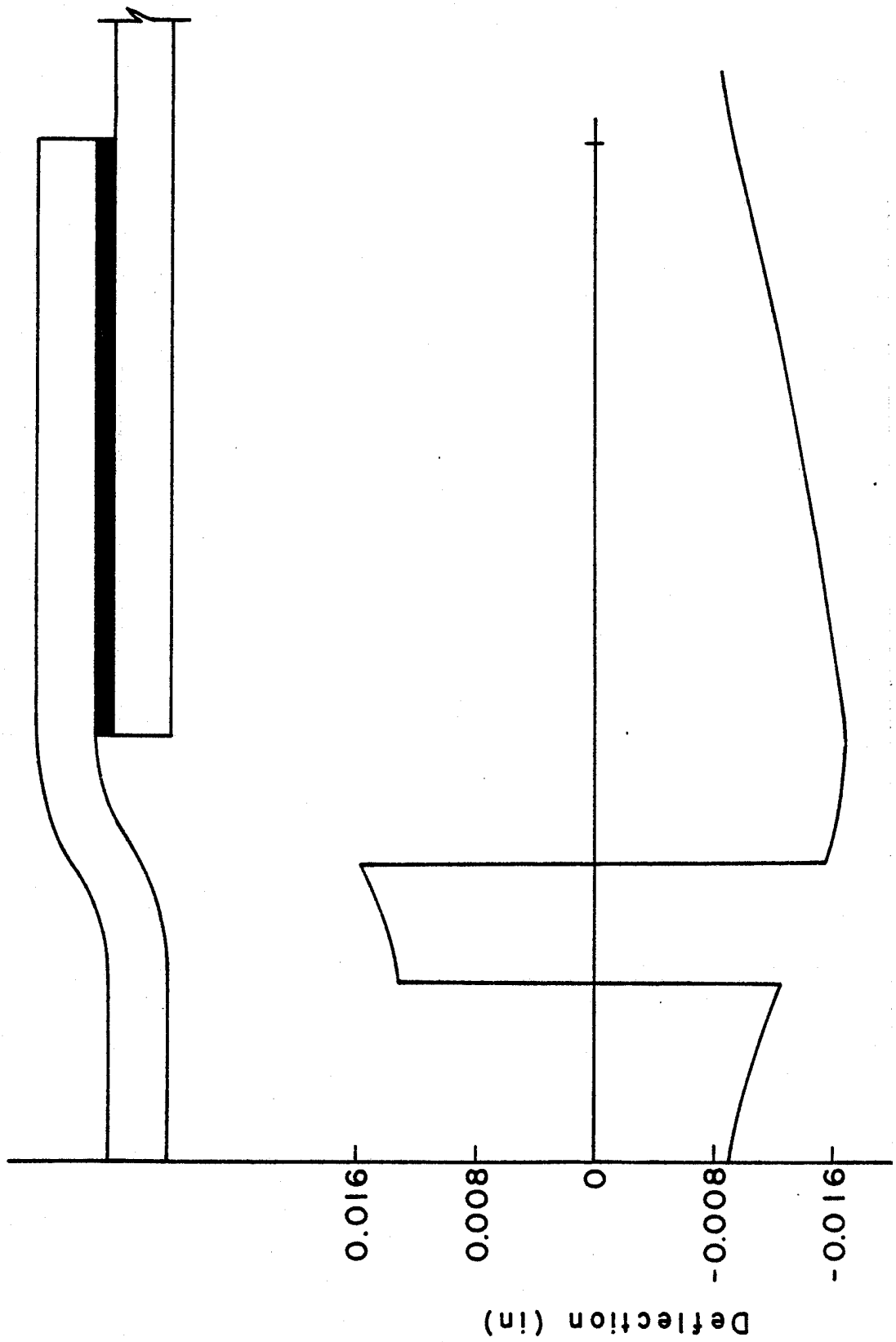


FIGURE 7: DEFLECTION OF NEUTRAL SURFACE AT FAILURE LOAD

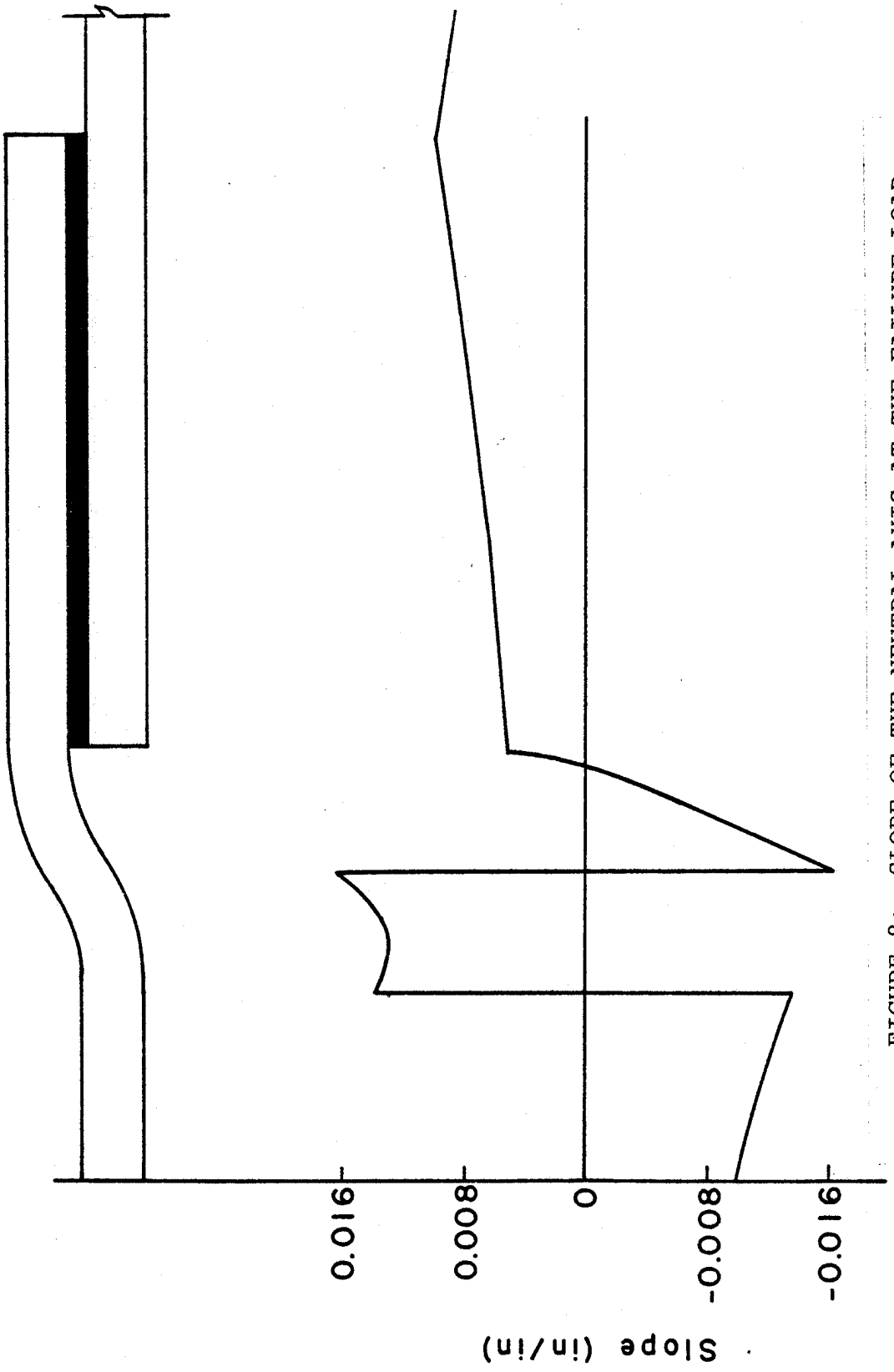


FIGURE 8: SLOPE OF THE NEUTRAL AXIS AT THE FAILURE LOAD

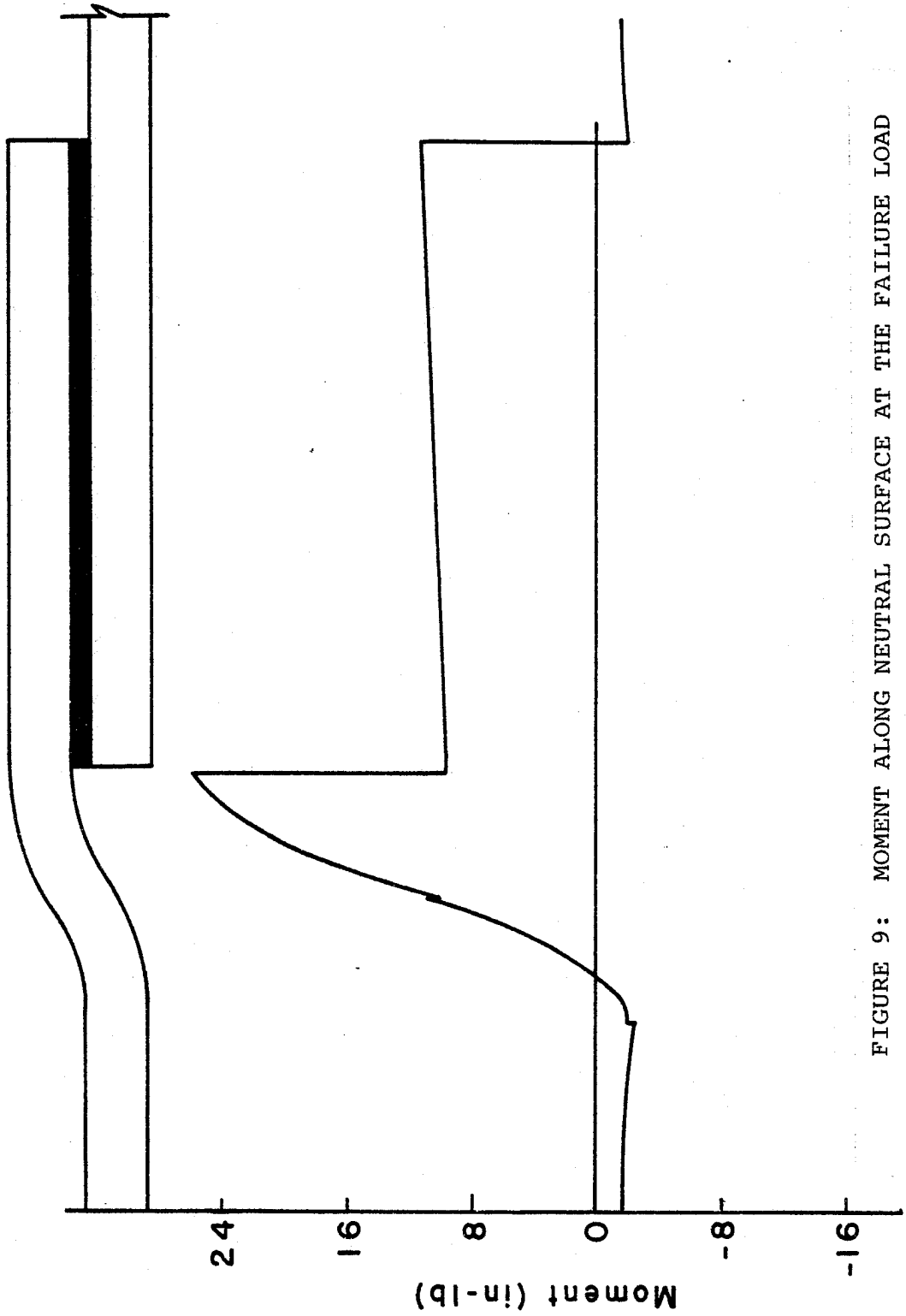


FIGURE 9: MOMENT ALONG NEUTRAL SURFACE AT THE FAILURE LOAD

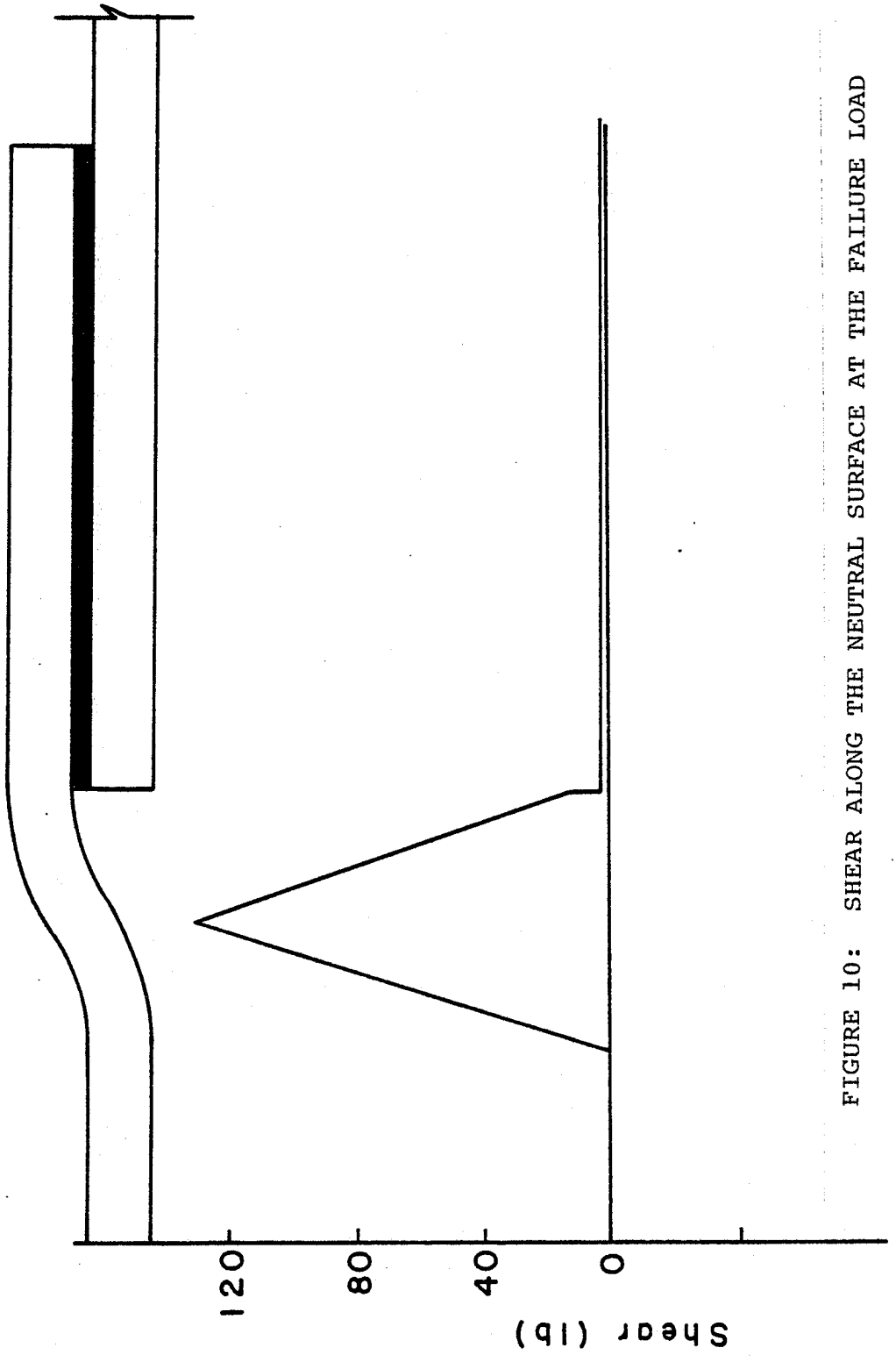


FIGURE 10: SHEAR ALONG THE NEUTRAL SURFACE AT THE FAILURE LOAD

joint behavior is to investigate the response of the joint to varying tensile loads. Figures 11 and 12 provide a clear indication of the deviation from linearity even for small values of load. Both the deflection (Figure 11) and moment (Figure 12) were recorded at the beginning of SEG3. (i.e. $S_3 = 0$)

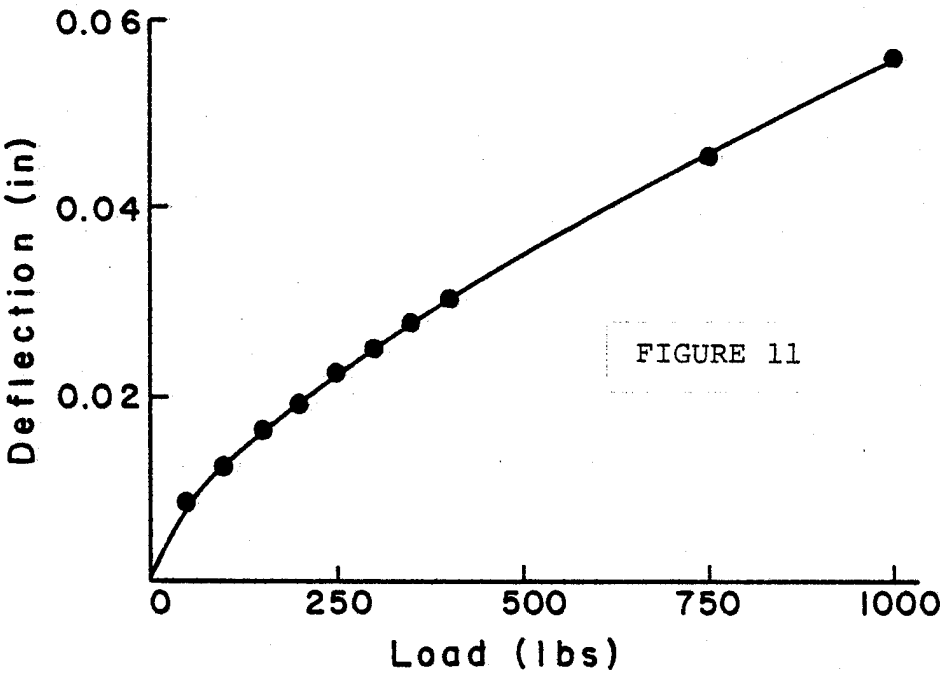


FIGURE 11

RESPONSE OF THE JOGGLE-LAP JOINT AT $S_3 = 0$

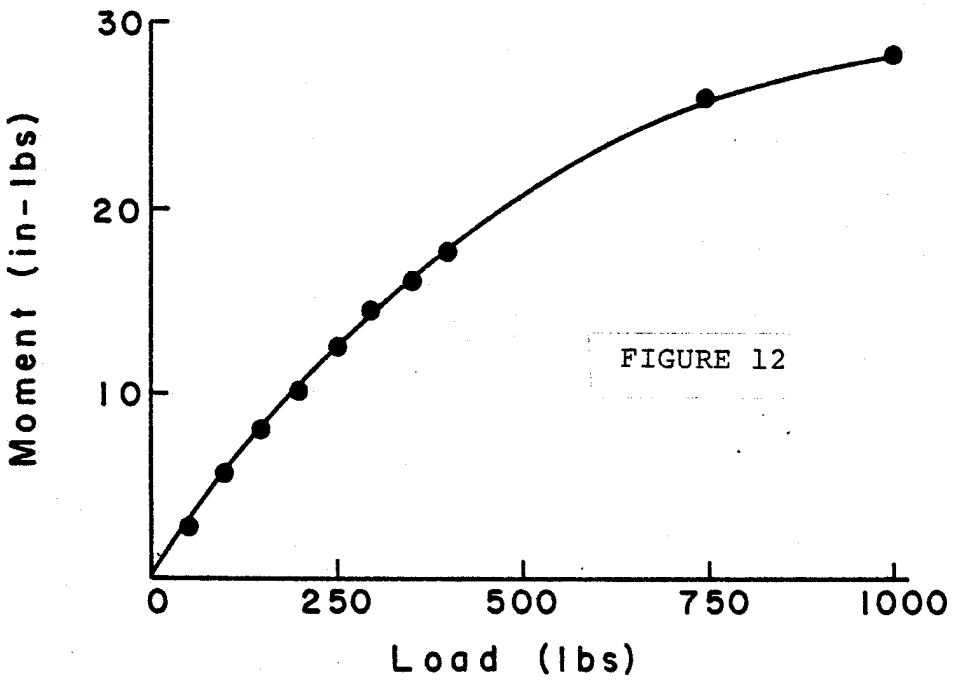


FIGURE 12

b. Finite Element Model (tension)

Anticipating the shortcomings of a beam bending model in the adhesive zone, defined to be the area of actual bonding, it was decided to model this area using finite-element methods. One of the underlying assumptions of small deflection beam theory is that plane sections remain plane during pure bending action. Clearly the validity of this assumption is questionable in the bonded area. Another reason for employing the finite element technique was to uncover any local stress concentrations that may not be revealed in a beam analysis. The finite-element mesh, consisting of 7 material types, is shown in Figure 13. Boundary conditions in the form of concentrated loads were applied to each of the finely meshed ends. Loading conditions were applied away from the adhesive layer at a distance of 1.5 times the thickness in an effort to minimize the effects of the end loads upon the stress solution. An explanation of how these boundary conditions were determined will follow shortly. A plane stress analysis was utilized to calculate the displacement and stress fields. Figures 15 through 17 are the result of a plotting routine which displays lines of constant stress. The figures should be interpreted in the same manner as that of a topographical map. Adjacent lines spaced closely together indicate areas of high stress gradients and possible sites for structural failure. The

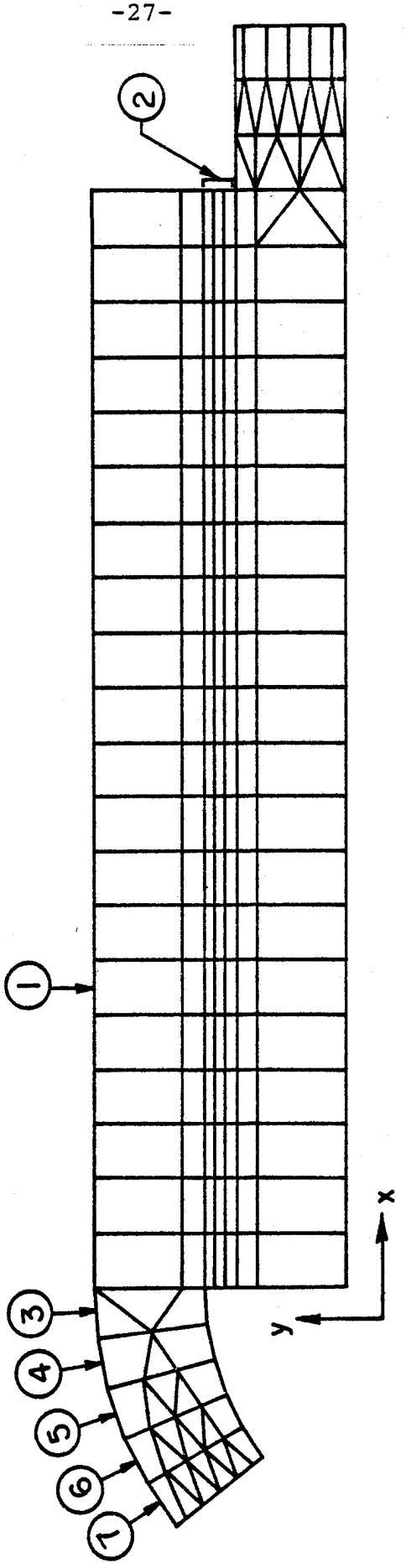


FIGURE 13: FINITE ELEMENT MESH OF THE ADHESIVE ZONE

7 Material Types

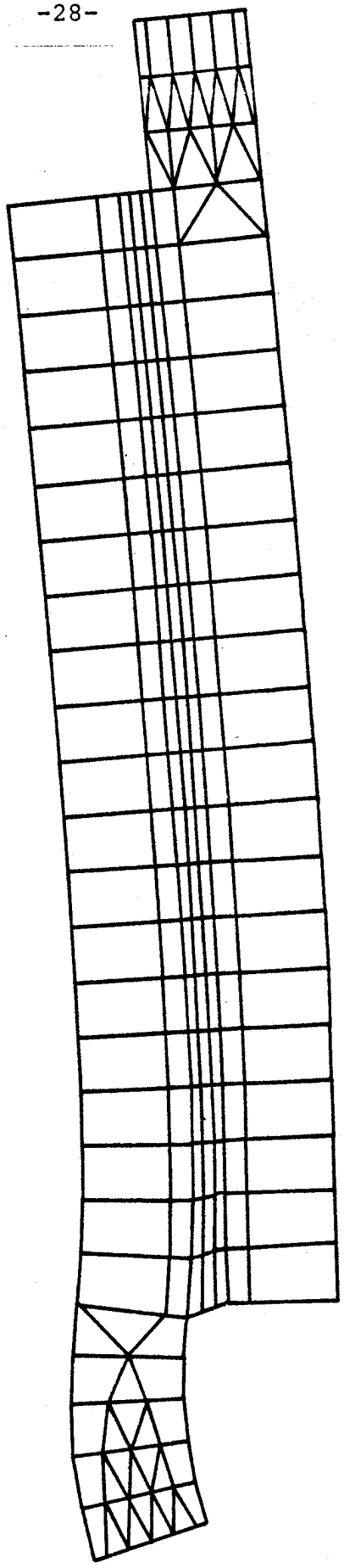


FIGURE 14: DEFORMED MESH AT THE FAILURE LOAD

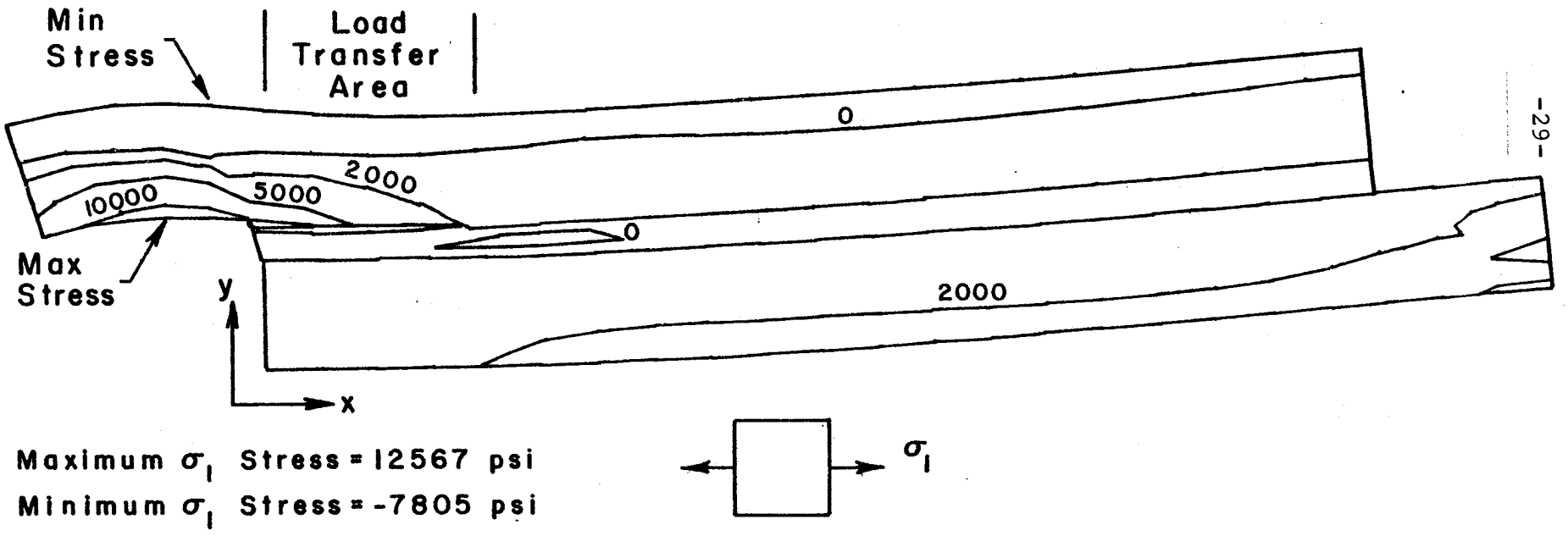


FIGURE 15: THE ADHESIVE ZONE IN TENSION - CONTOURS OF σ_1 STRESS

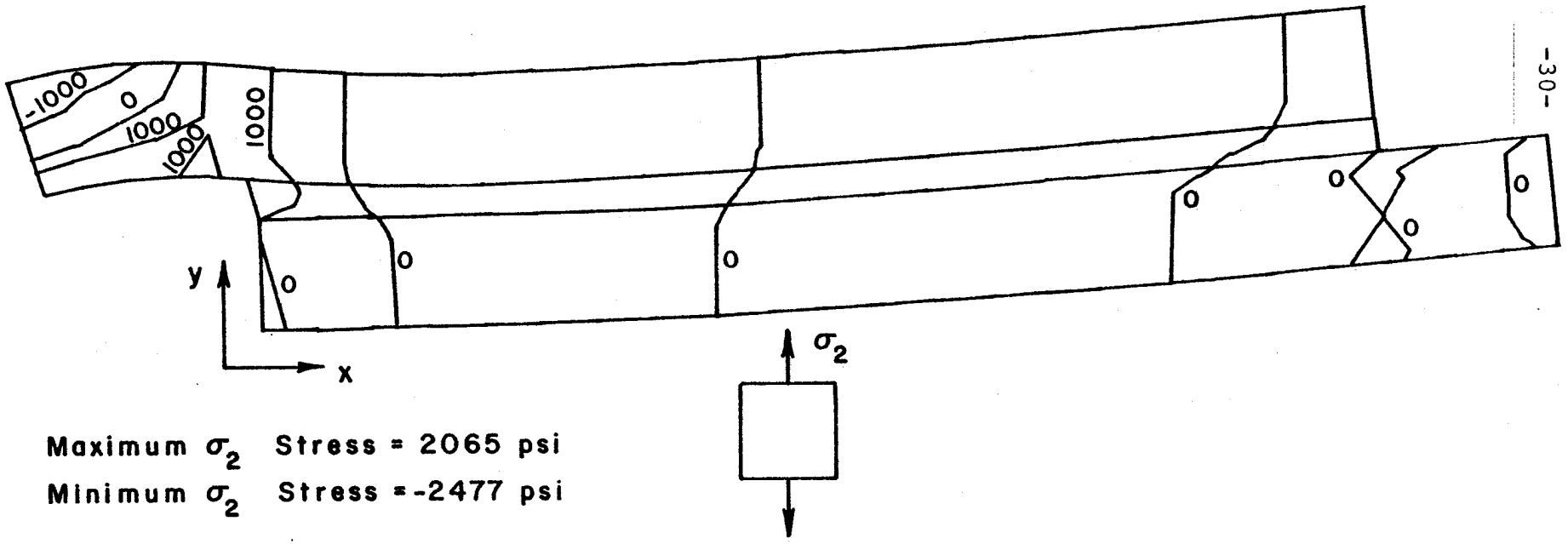


FIGURE 16: THE ADHESIVE ZONE IN TENSION - CONTOURS OF σ_2 STRESS

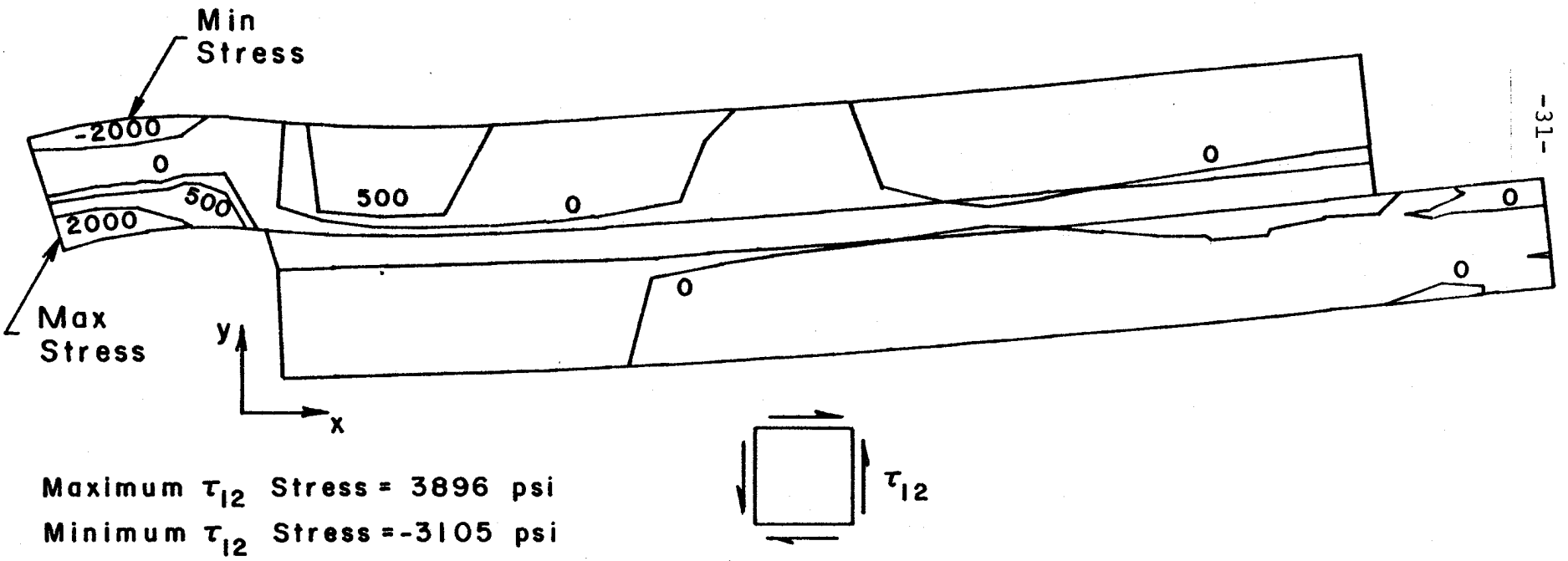


FIGURE 17: THE ADHESIVE ZONE IN TENSION - CONTOURS OF τ_{12} STRESS

figures are labeled according to the component of stress being displayed. All three plots are the result of loading the specimen at the tensile failure load and are representative of the deformed geometry.

The limitations of the beam bending model are clearly displayed in Figure 15 and reveal the justification for the finite-element model. Shown in the figure is a smooth transition of stress across a change in cross-sectional area, (i.e. shift of the neutral axis) as calculated by the finite-element method. Experimental results have shown this to be a correct representation of the stresses. Beam analysis would have shown a sharp discontinuity in the stress profile where such a shift in the neutral axis occurs. Since the moment is nearly constant throughout SEG4 (see Figure 9) beam analysis would calculate σ_1 stress contours parallel to the adhesive layer. The σ_1, σ_2, and τ_{12} stress components are global oriented stresses as opposed to those that can vary according to element orientation. Marked on each figure are those areas where the assumptions made via beam analysis quite appreciably affect the accuracy of a correct solution.

Many analyses of lap joints assume a condition of constant shear stress in the adhesive layer itself. This would indeed be the case if the adherents were infinitely stiff as compared to the adhesive and also if the existence of a load transfer area was prohibited. Shear stress data

from the finite element model is plotted in Figures 18 and 19 and the indication is clear that the shear stress is not a constant in the load transfer area. The case of constant shear stress found toward the center of the adhesive zone, however, reveals the linear nature of the displacement function through the adhesive thickness in this area.

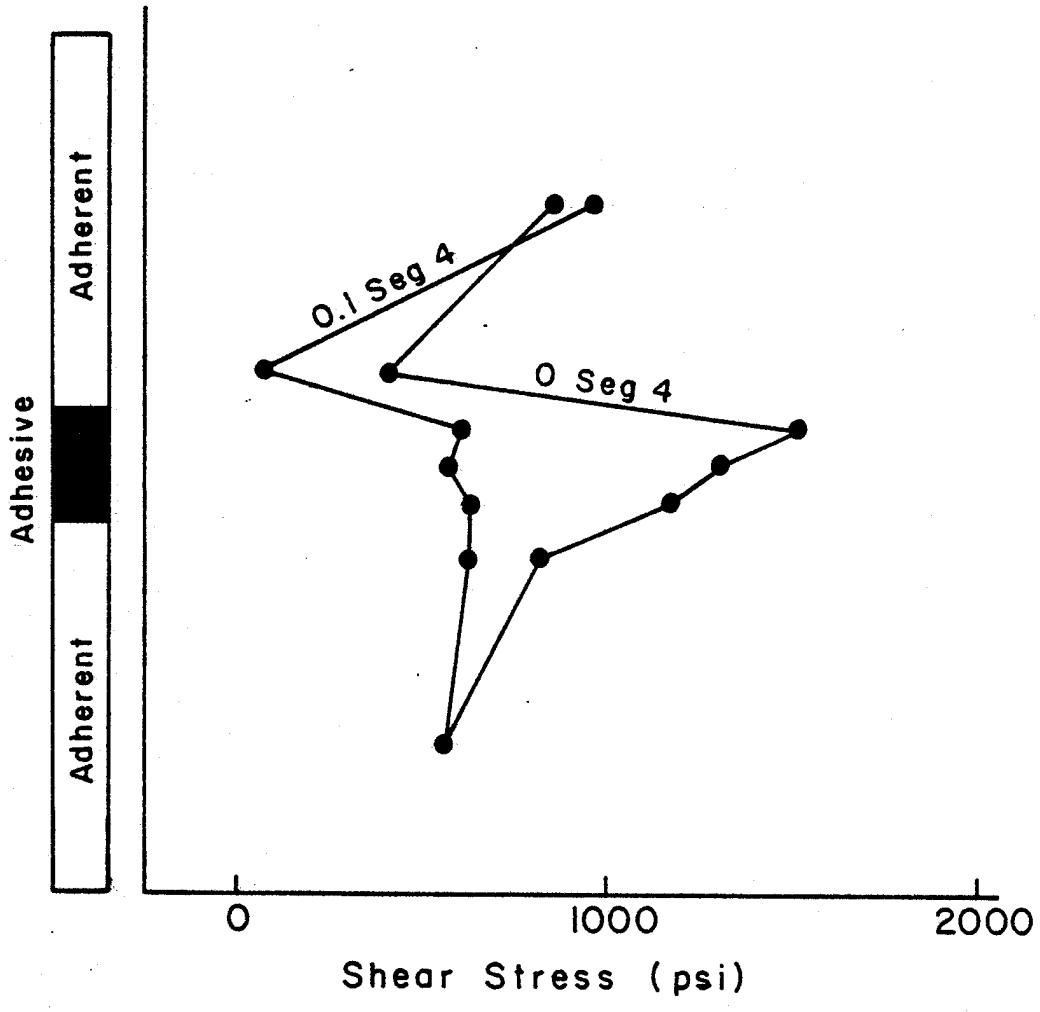


FIGURE 18: SHEAR STRESS VARIATION THROUGH THE THICKNESS OF THE ADHESIVE ZONE

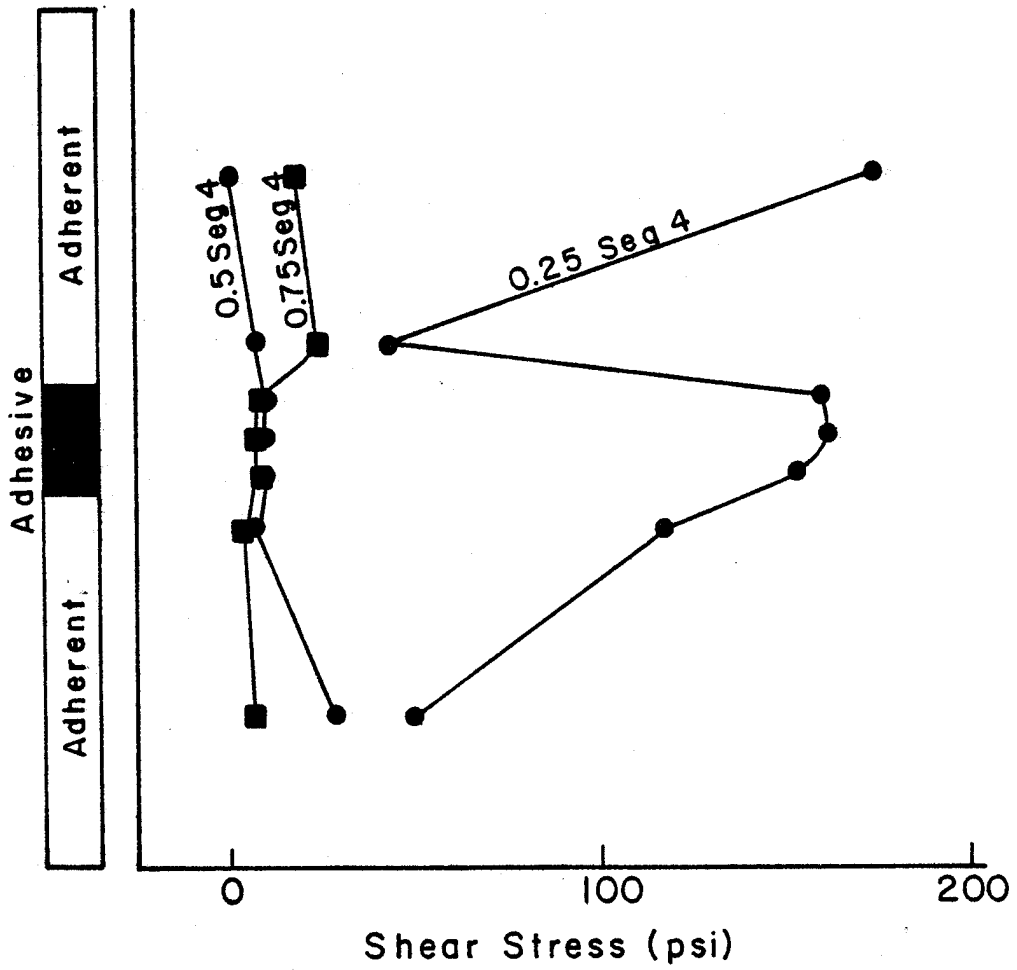


FIGURE 19: SHEAR STRESS VARIATION THROUGH THE THICKNESS OF THE ADHESIVE ZONE

Boundary Conditions for the Finite-Element Model

The boundary conditions for the finite-element model are determined by applying the stress distribution as directed by the beam bending model to the finely meshed ends of the undeformed geometry of the finite element model. In other words, the stresses in the deformed geometry (beam model) must be moved through a distance to their equivalent point of application in the undeformed geometry (finite-element model). The reason for this difficulty with boundary conditions is that we are currently utilizing a linearized finite element routine, SAP V[2], to solve a non-linear problem. Justification of such a procedure will hopefully become lucid with time.

To facilitate the derivation of a transformation routine, Figures 20 and 21 illustrate the following sign conventions. Figure 20 depicts a stress distribution for the left hand face of the finite element model with tension being taken as positive and compression being negative. Note that the neutral axis is not coincident with the centroidal axis inherent in the analysis of a curved beam. As mentioned previously, this fact yields a hyperbolic stress distribution which slightly complicates the computations. (SEE derivations of governing equation for stresses in a curved beam, Appendix A)

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[2]Structural Analysis Program V; University of Southern California, Department of Civil Engineering, Oct. 77.

FIGURE 20

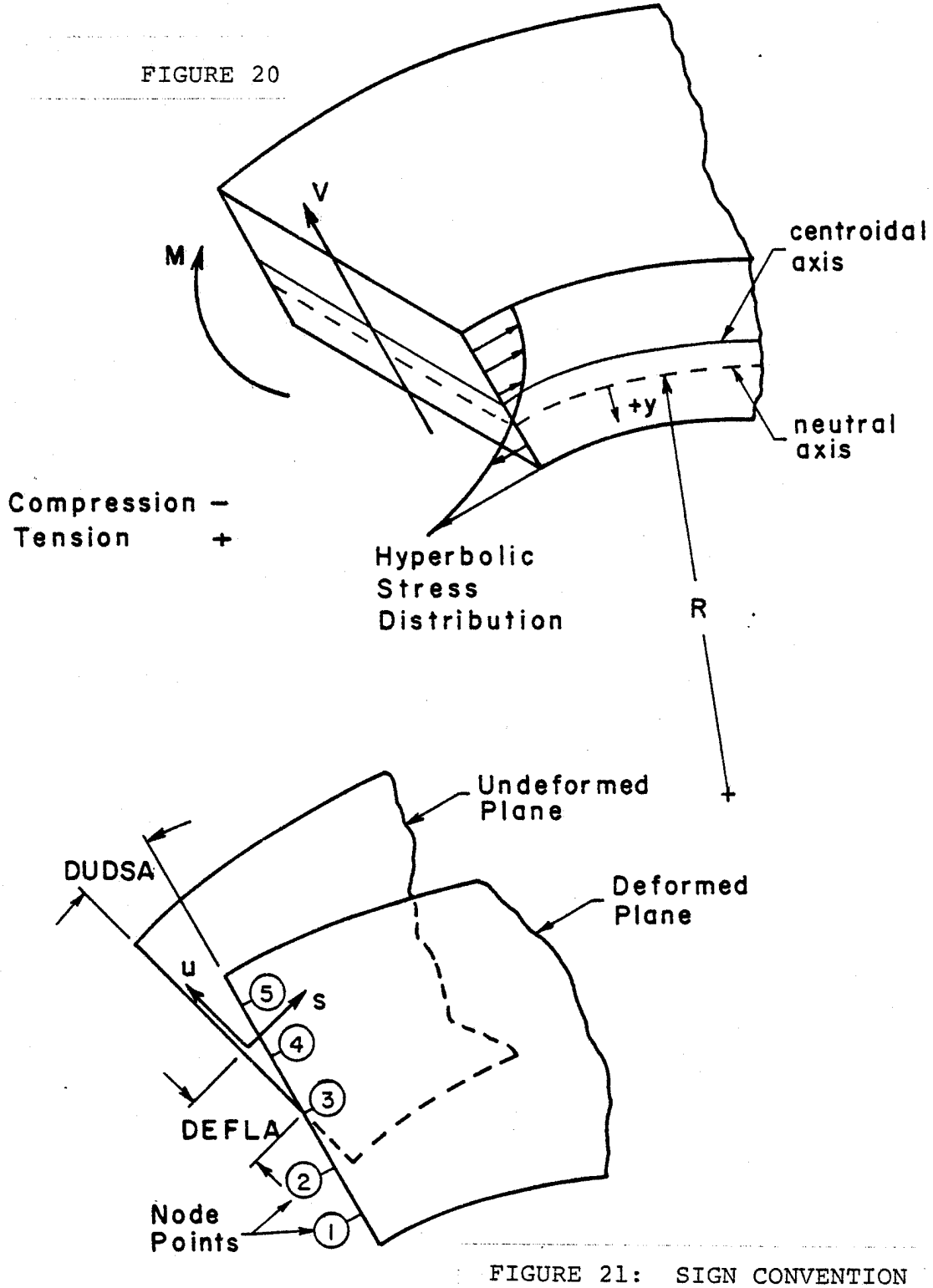

FIGURE 21: SIGN CONVENTION

Figure 21 reveals a planar view of the deformed and undeformed sections. It is assumed in this derivation that the section of the beam can at most undergo a translation and a rotation. Translations are measured via the parameter DEFLA and are positive radially outward as shown. Small deflection theory also allows the rotations to be written as a change in slope. This parameter is DUDSA and is positive counter-clockwise.

With these sign conventions clearly in mind the stress distribution of the deformed geometry may now be resolved into concentrated force components. Representing the hyperbolic stress distribution as equivalent point forces and point couples acting at nodal points labeled 1 through 5 on Figure 21 corresponds mathematically to an integration of the stress distribution between fixed limits.

$$F_{ni} = \frac{M}{\bar{u}\, a} \int_{h_{i-1}}^{h_i} \frac{u}{R-u}\, du + \int_{h_{i-1}}^{h_i} F \cos(\theta + \text{DUDSA})\, du \qquad (6)$$

where $i = 1\text{-}5$

F_{ni} = nodal force component

M = moment

$\bar{u}$ = distance between neutral and centroidal axes

a = cross-sectional area

R = radius of curvature

F = load

θ = angle subtended by SEG3

DUDSA = local slope of deformed neutral axis

The first term of Eq. (6) represents the contribution from the hyperbolic stress distribution. The second term acts to superimpose the component of force due to longitudinal loading.

A correcting moment is calculated for each node to equilibrate the two representations of stress on the section.

$$M_{corr} = \int_{h_{i-1}}^{h_i} \sigma(u)u\, du - F_{ni}u \qquad (7)$$

The need for the correcting moment is due to the fact that a distributed force is now represented by a point force as shown in Figure 22.

The next step follows from a translation of the point forces. Elementary statics dictates that a point force may be equivalently represented by the same point force and an added moment to account for the translation from the original line of action.

After carrying out a similar procedure for the stresses at the right hand side of the finite element model, the entire system is set in equilibrium by accounting for the shear acting on each face of the model. The values of shear are obtained directly from the beam bending model. Thus a correct set of boundary conditions has been determined for the finite-element model of the adhesive zone. A computer routine designated by CONVERT was written to calculate appropriate boundary conditions and may be found

in Appendix C.

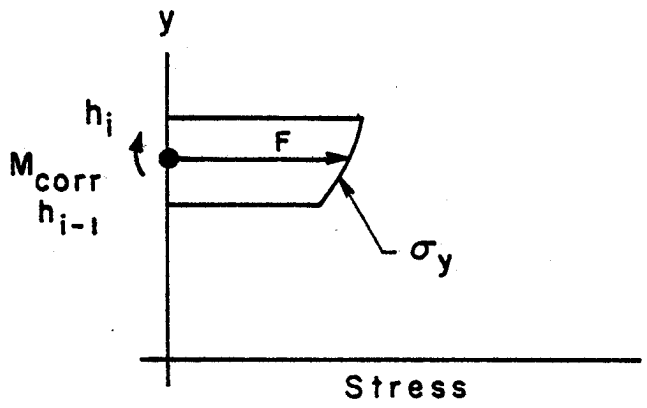

FIGURE 22: ILLUSTRATION OF THE CORRECTIVE MOMENT

Methods of Analysis

B. Flexure Loading

a. Beam Model

The bending behavior of the "joggle-lap" joint was also studied. It was found that the theoretical analysis was far simpler than that encountered for tensile loading. Each segment of the joint (see Figure 6) was modeled as if it were in pure bending. Stresses in the straight beam numbers were calculated via the flexure formula while for the curved beams the formula

$$\sigma_y = \frac{My}{(R-y)\bar{y}a} \tag{8}$$

where M = moment

y = coordinate from the neutral surface (positive radially inward)

R = radius of curvature

$\bar{y}$ = distance between centroidal and neutral axes

a = crossectional area

was used.

In order to compute bending stresses in SEG4 (layered beam) it is necessary to introduce the notion of equivalent sections. In this method we assume all materials to have the same modulus of elasticity. By replacing the actual section with a mechanically equivalent one allows

the flexure formula to be used as a means of computing stresses. The width of the sections are varied so that the new width equals the ratio of the old modulus of the material to the new modulus of the material times the old width as shown in Figure 23. Computing $I_{eq.}$ for the specimen geometry,

$$Ieq = \sum_{i=1}^{3} \left(\frac{1}{12} b_i h_i^3 + a_i d_i^2\right)$$

b = length of base

h = length of side

a = area

d = distance between element neutral axis and overall section neutral axis

it is apparent that the effect of the adhesive layer on overall section stiffness is negligible. Using the flexure formula and the relation

$$(\sigma_x)_{actual} = \frac{E_{old}}{E_{new}} (\sigma_x)_{equiv.}$$

the stresses in SEG4 may easily be calculated.

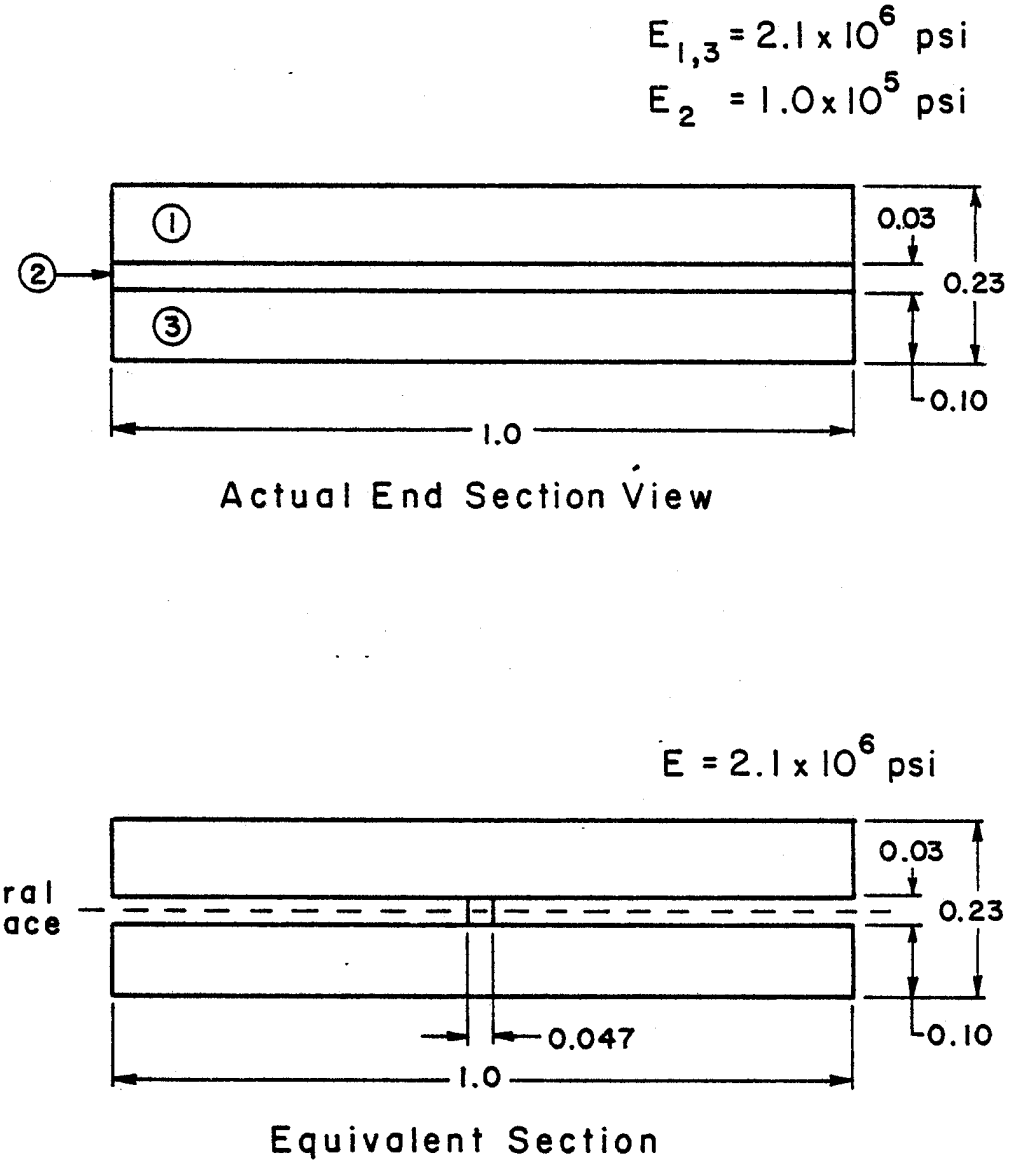

FIGURE 23: METHOD OF EQUIVALENT SECTIONS

b. Finite Element Model (flexure)

The boundary conditions of the finite-element model may be changed to accommodate pure bending. By utilizing couples at the finely meshed ends of the model, stresses in the adhesive zone may be monitored where it has been shown that the results from beam theory are less accurate. Figures 24 through 26 display σ_1,σ_2, and τ_{12} stress contours respectively within the "joggle-lap" joint in pure bending.

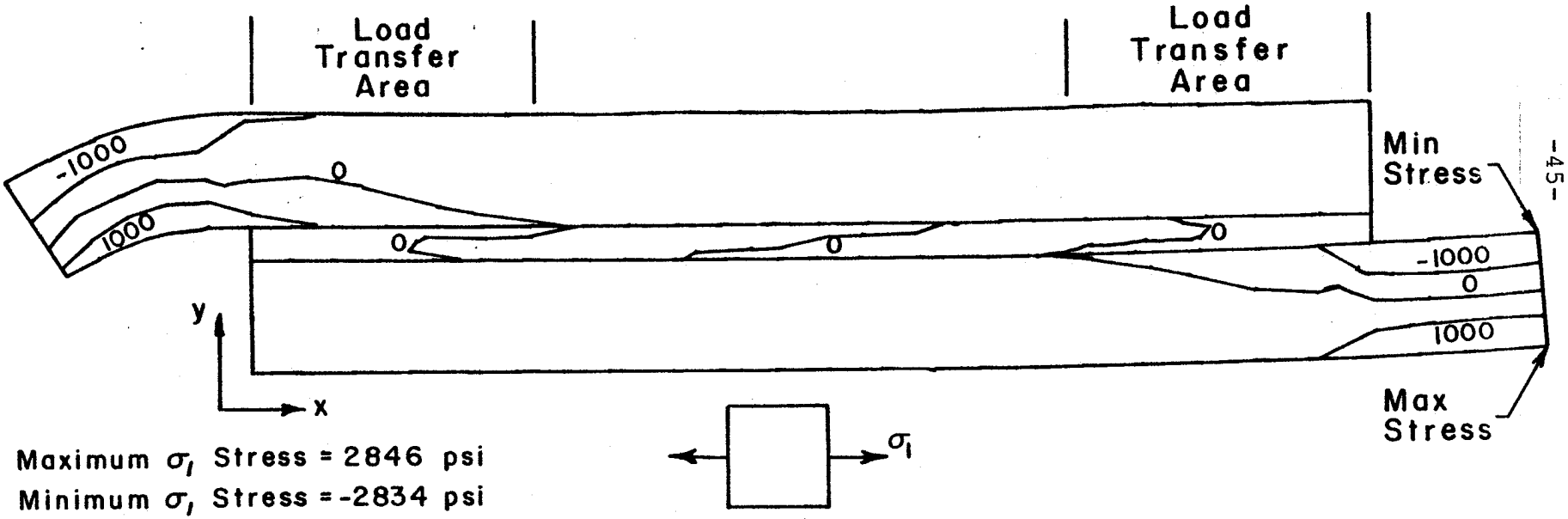

FIGURE 24: THE ADHESIVE ZONE IN BENDING - CONTOURS OF σ_1 STRESS

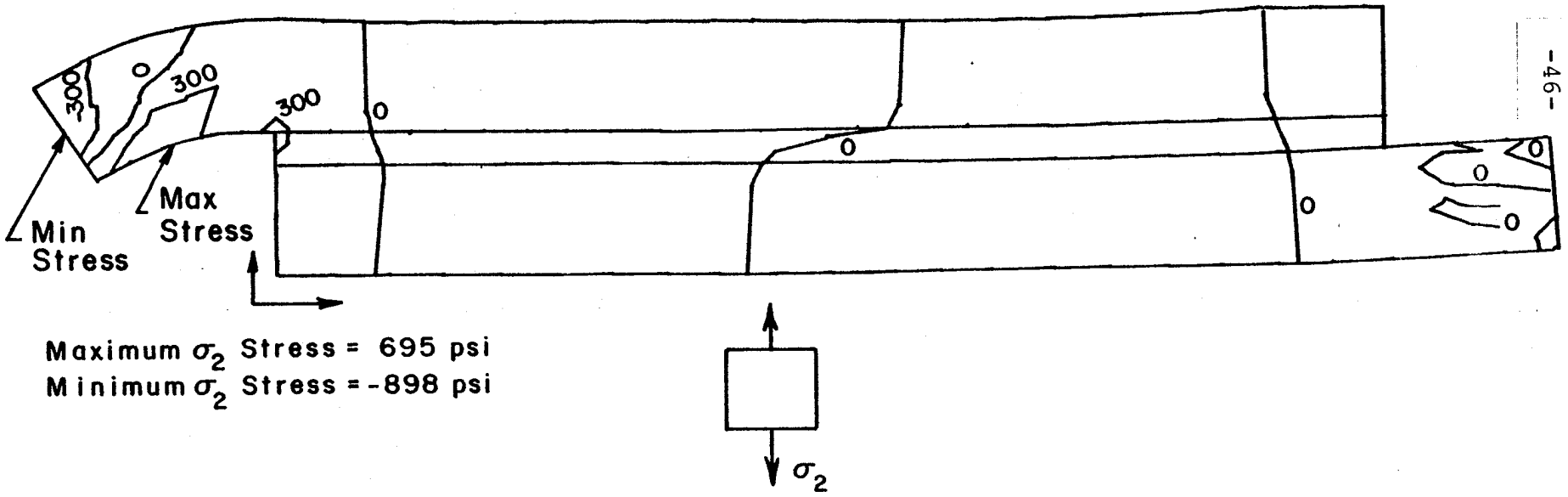

FIGURE 25: THE ADHESIVE ZONE IN BENDING - CONTOURS OF σ_2 STRESS

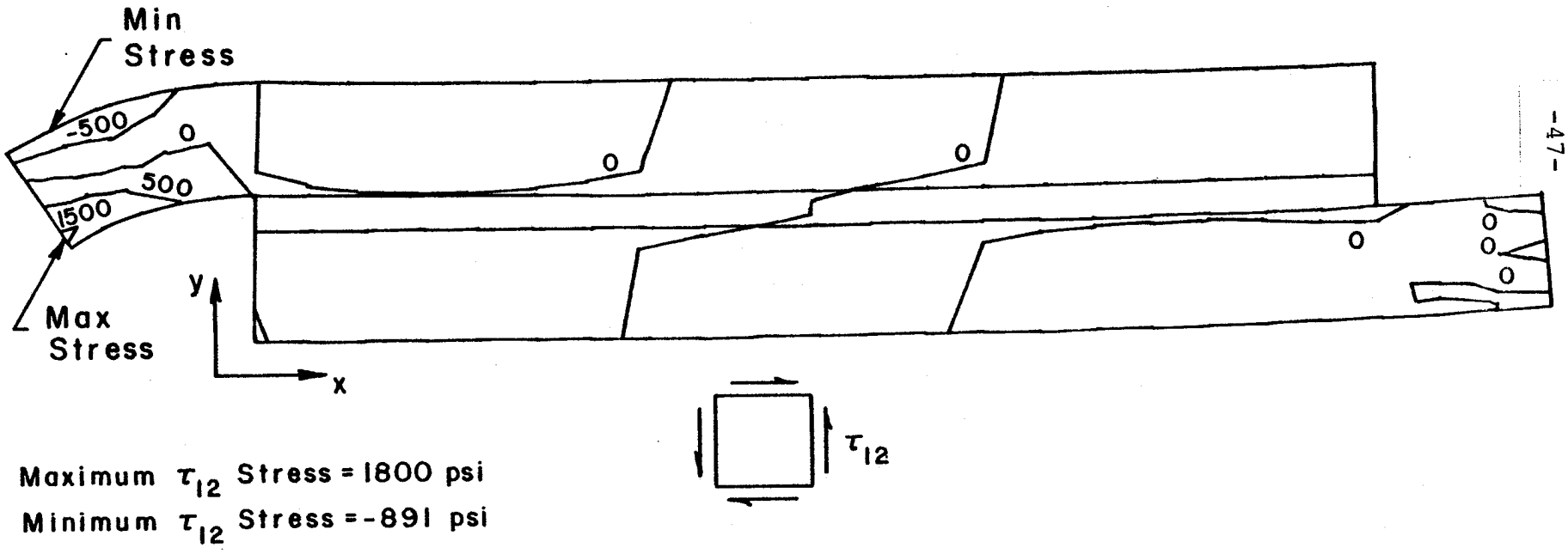

FIGURE 26: THE ADHESIVE ZONE IN BENDING - CONTOURS OF τ_{12} STRESS

IV. Experimental Results

A. Tension

As set forth in the objectives of such a study, an emphasis was to be placed upon developing joint geometries which will accommodate high rate fabrication techniques. In an effort to meet this criterion experimentally, it was necessary to utilize a joint configuration currently being molded in industry. The time and expense of developing in-house molding capabilities proved to be beyond the scope of the research at hand. Thus, test sections were cut from premolded panels of SMC which were later bonded together to form the joint.

The bonding operation was also directed toward high fabrication procedures. All test specimens were adhesively joined at Goodyear Adhesives Division, Ashland, Ohio, via production adhesives application techniques. It was felt that by using these sophisticated application procedures optimum adhesive properties could be obtained.

In general, SMC is defined to be an anisotropic material because of the substantial difference between in-plane and out-of-plane properties. Referring to the coordinate system of Figure 1, the constitutive relations

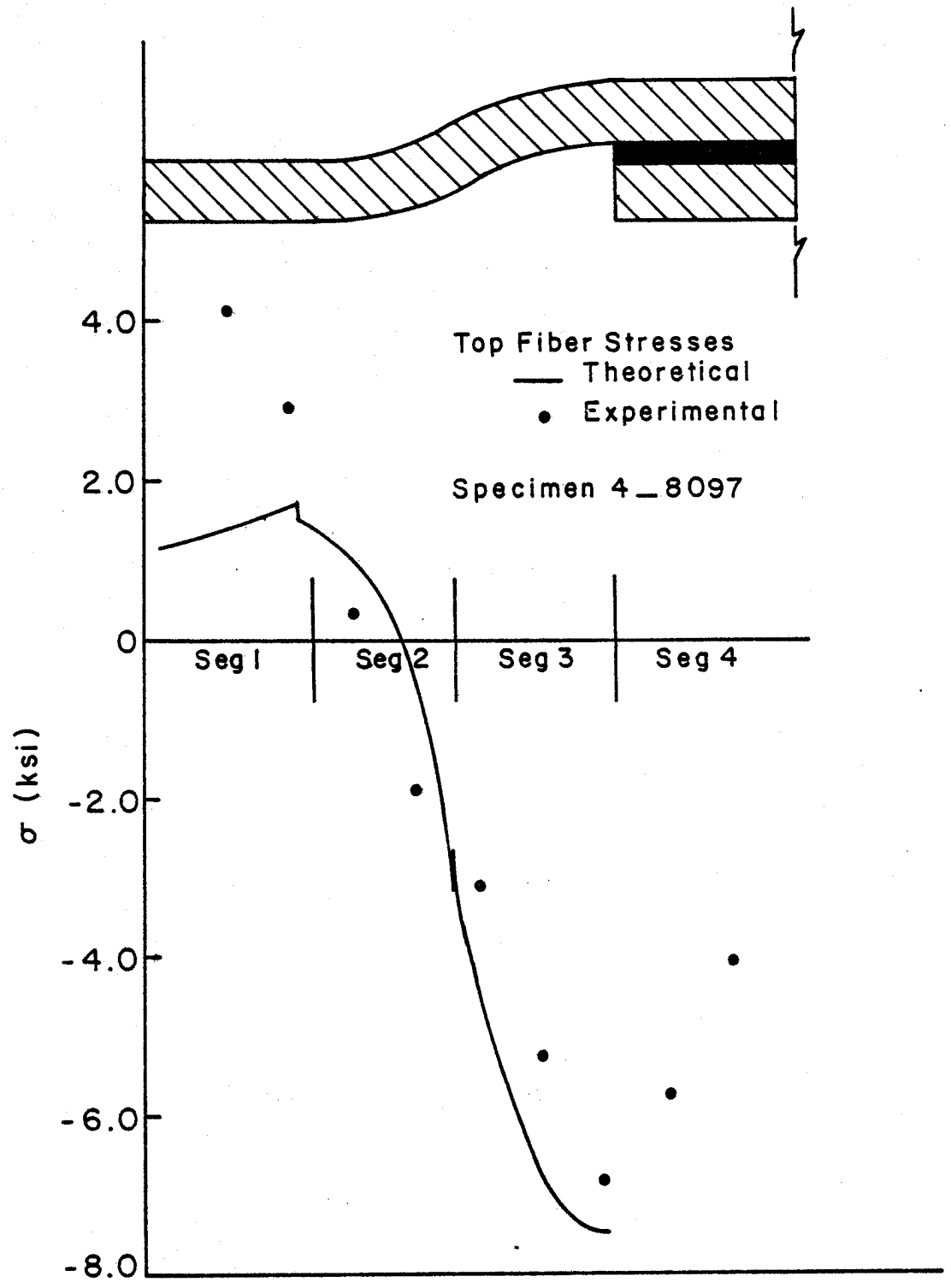

FIGURE 27: ELASTIC RESPONSE DUE TO TENSION - TOP FIBER STRESSES

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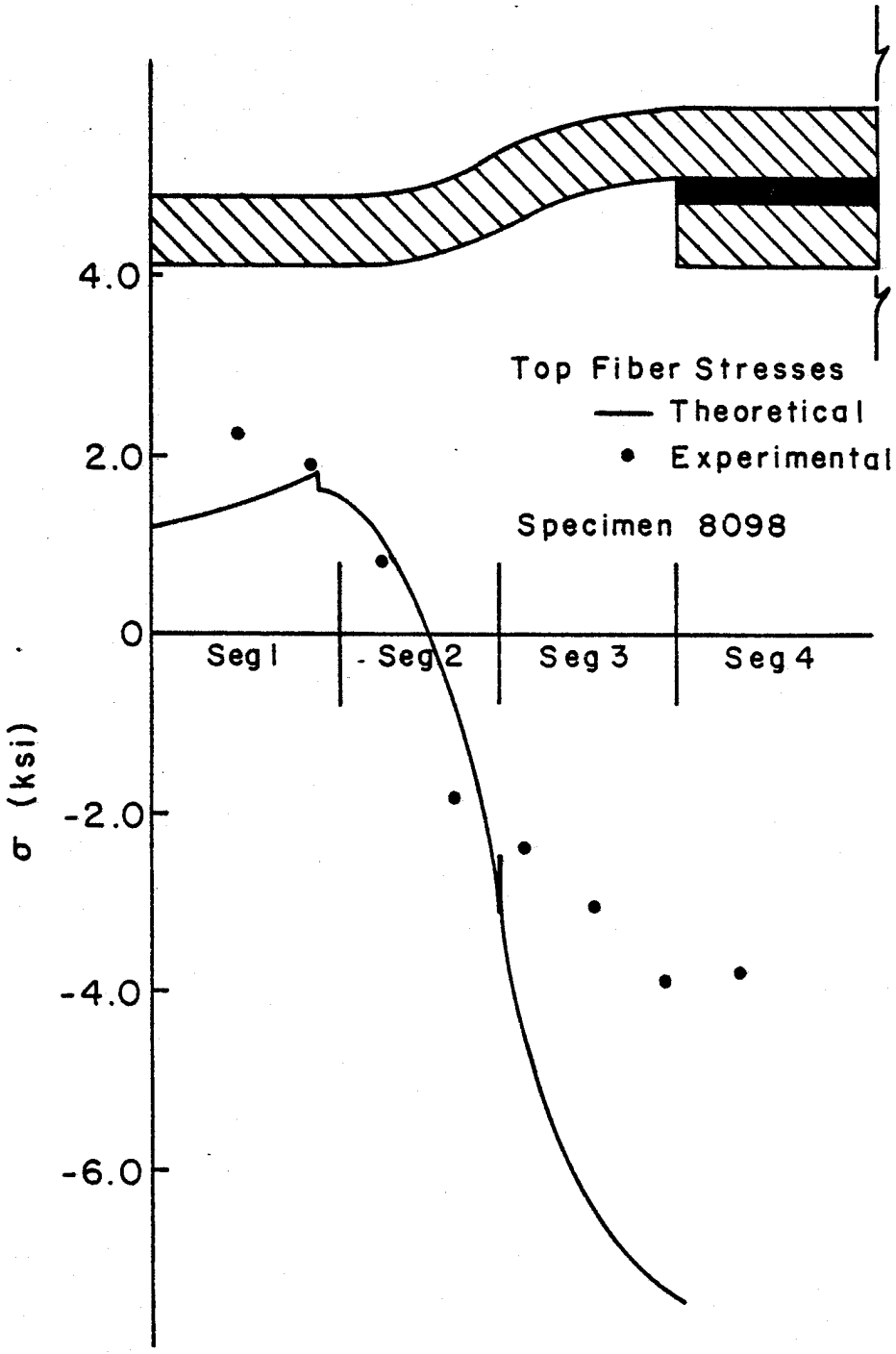

FIGURE 28: ELASTIC RESPONSE DUE TO TENSION - TOP FIBER STRESSES

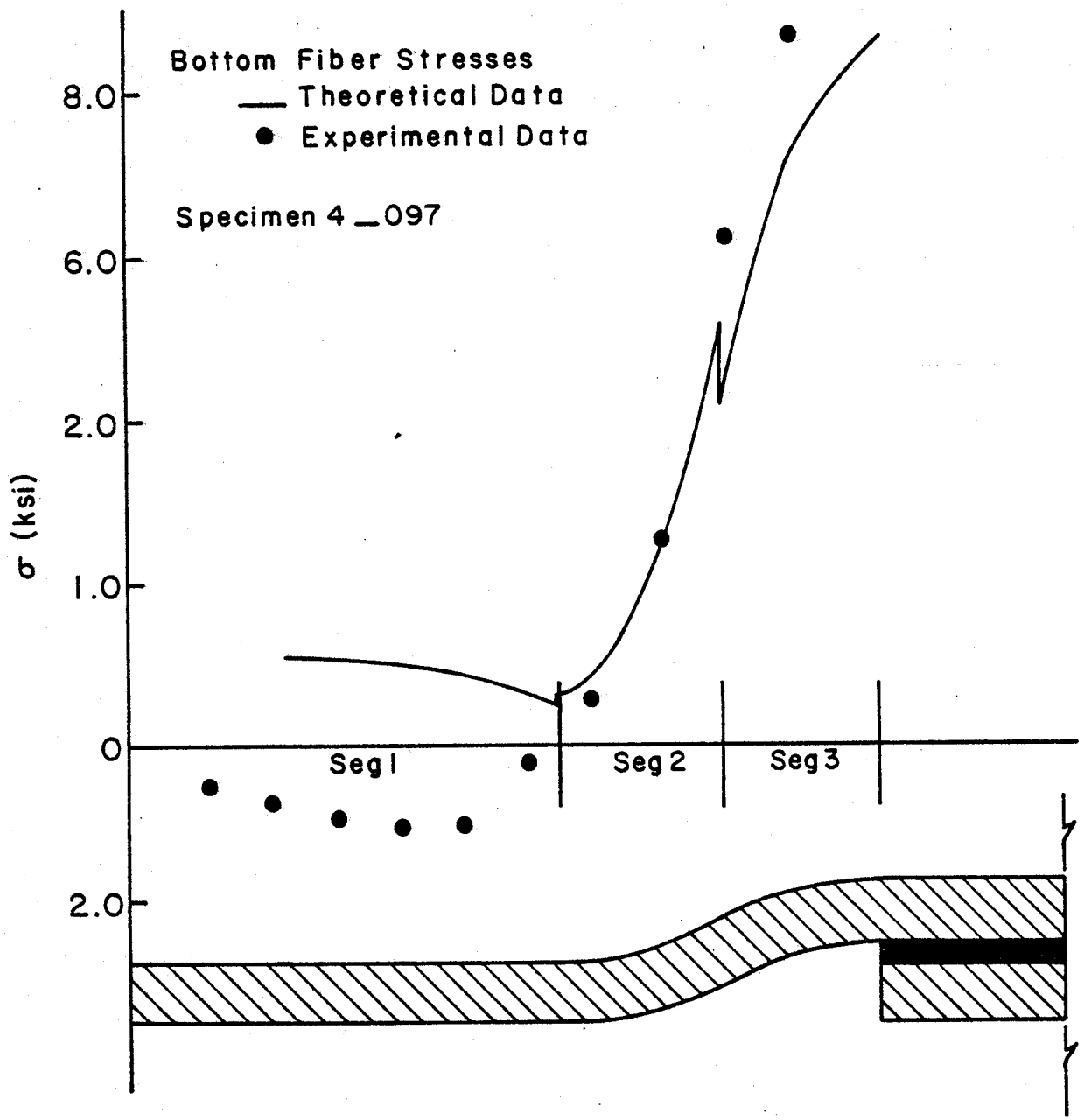

FIGURE 29: ELASTIC RESPONSE DUE TO TENSION - BOTTOM FIBER STRESSES

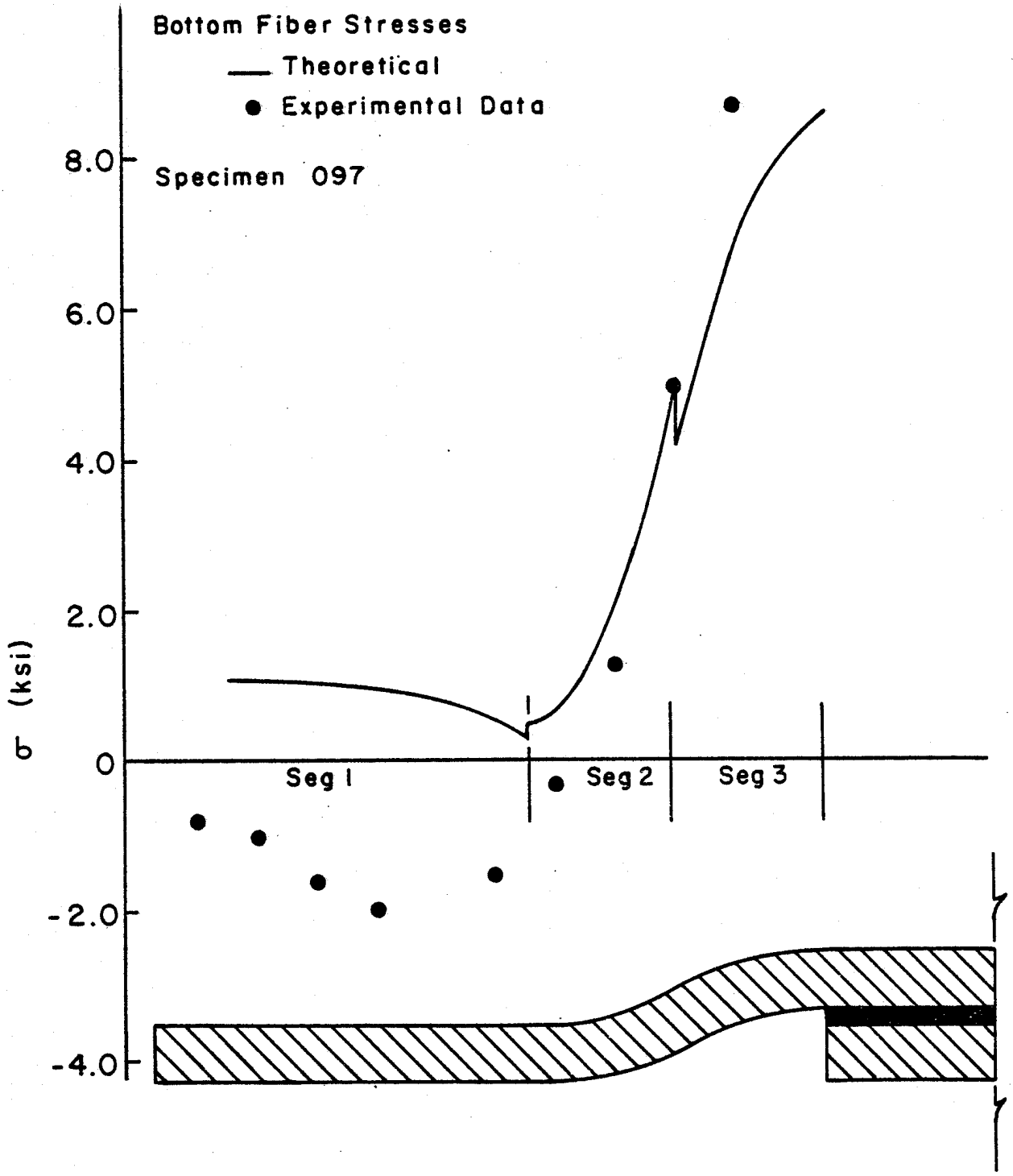

FIGURE 30: ELASTIC RESPONSE DUE TO TENSION - BOTTOM FIBER STRESSES

that the joint invariably strained beyond the small-deflection range at considerably small loadings. It was therefore a rather arduous task to approximately determine the experimentally applied moment to the joint. The correlation between the theoretical and experimental data may be referenced in Figures 31-34. As in the case of tensile loading, it should be noted that the stresses in SEG1 are again considerably higher than those predicted by theory, which is attributable to the molded geometry.

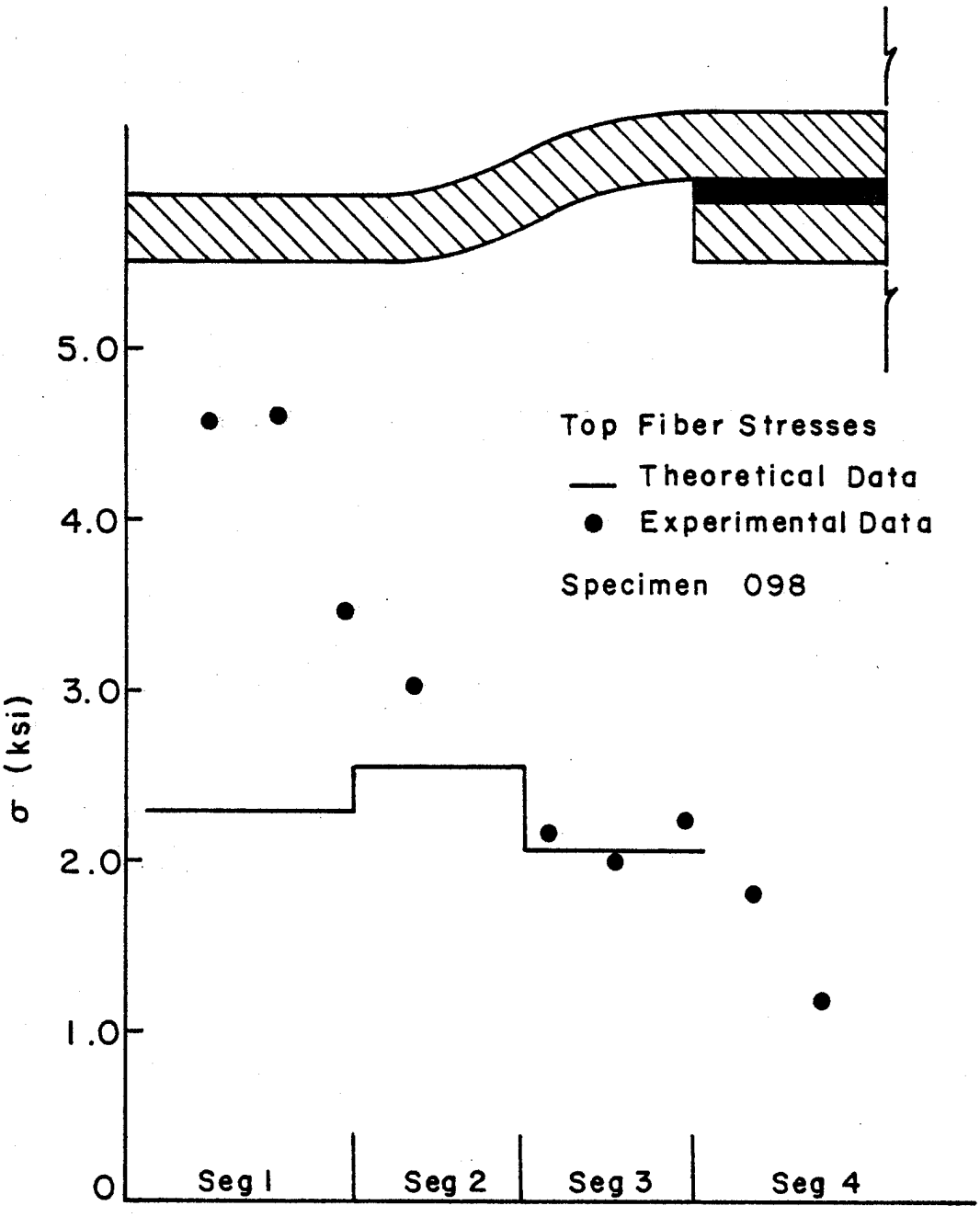

FIGURE 31: TOP FIBER STRESSES DUE TO BENDING

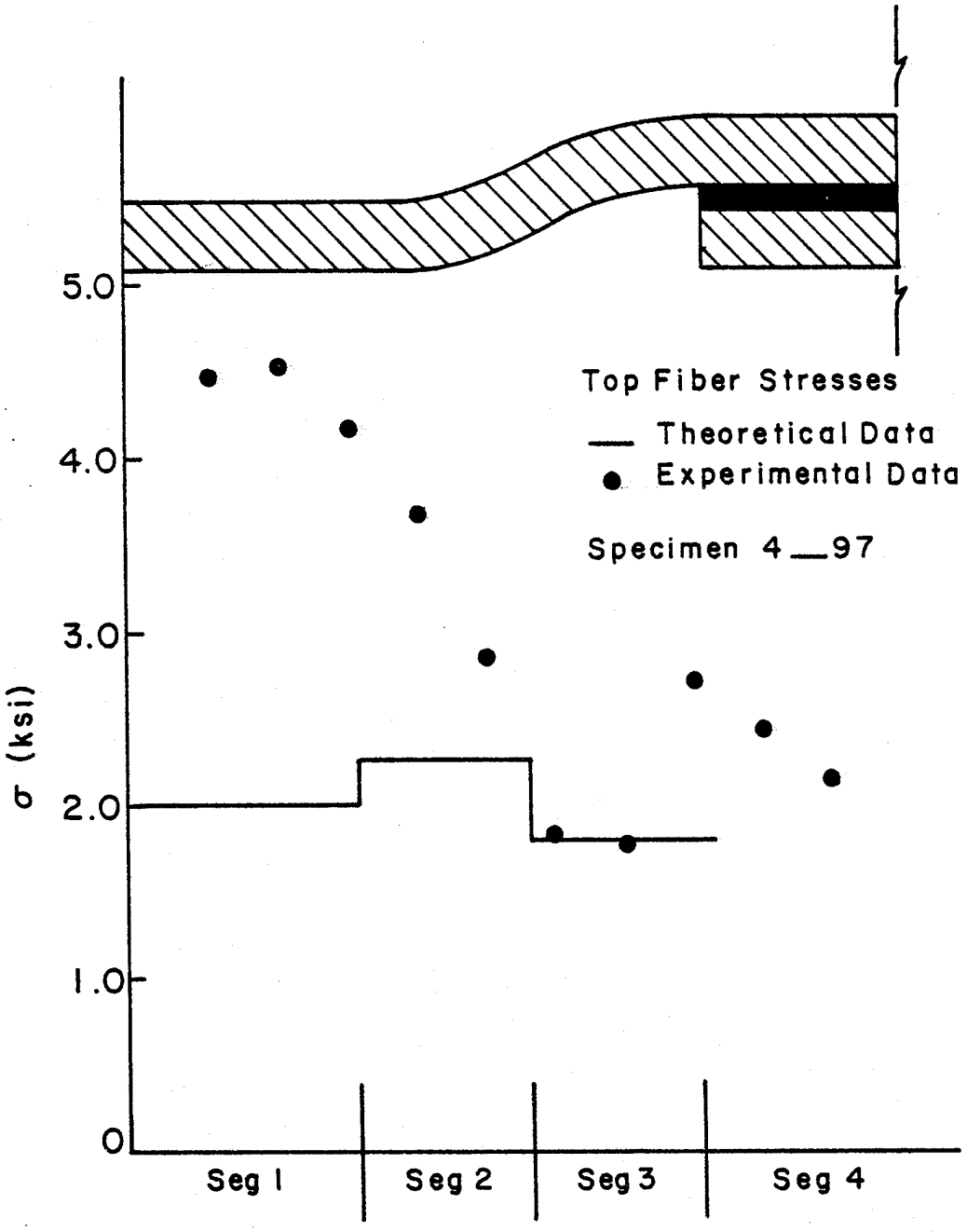

FIGURE 32: TOP FIBER STRESSES DUE TO BENDING

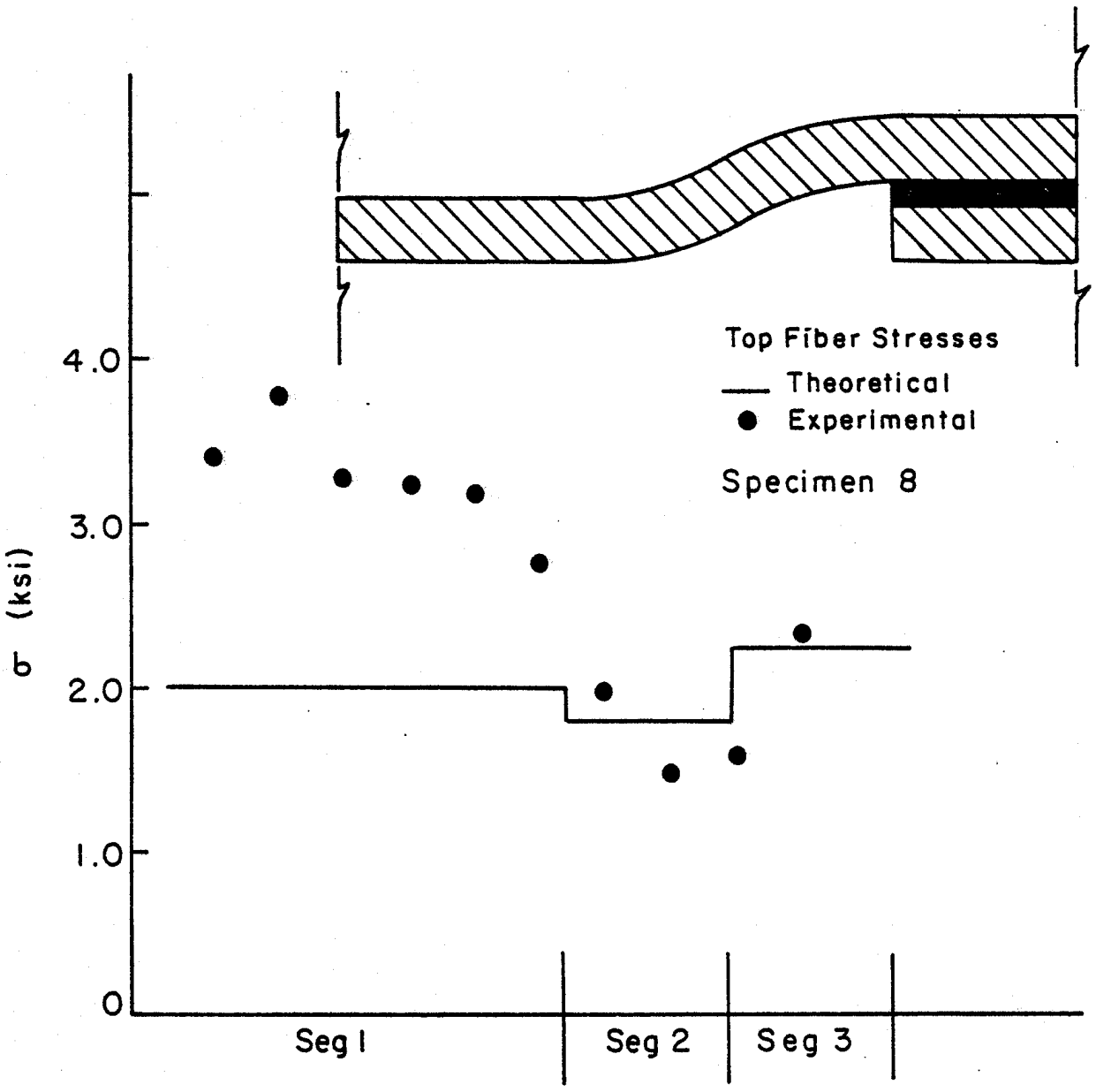

FIGURE 33: BOTTOM FIBER STRESSES DUE TO BENDING

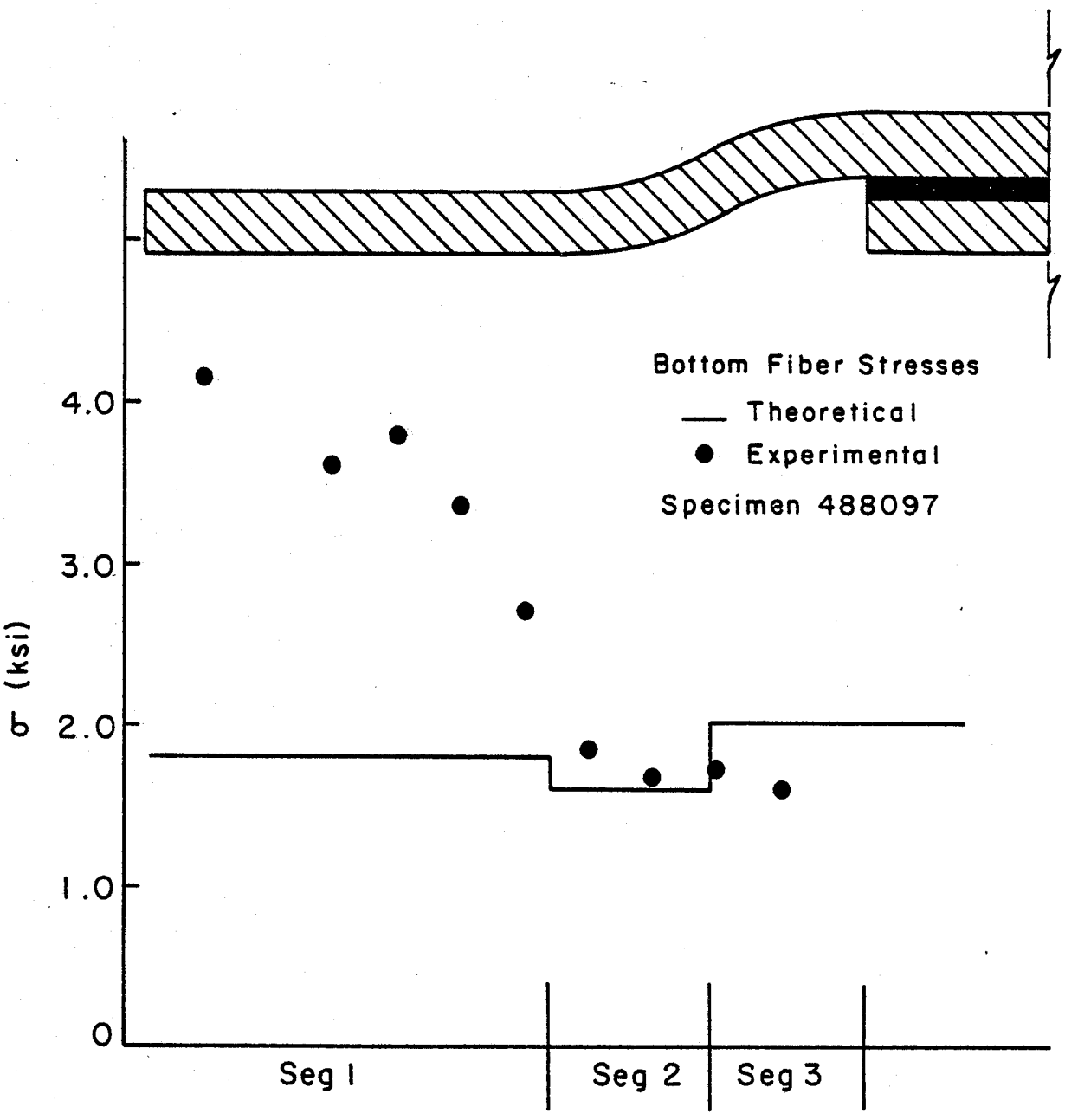

FIGURE 34: BOTTOM FIBER STRESSES DUE TO BENDING

Table 3

Experimental Results from Tension Tests

Specimen #	Contact Area (in^2)	Loading Condition	Failure Load (LBS)	Failure Mode*
4__097	1	tension	191	flexure
4_8097	1	tension	206	flexure
4___98	1	tension	209	flexure
4__098	1	tension	171	flexure
___97	1.75	tension	169	flexure
___98	1.75	tension	242	flexure
__097	1.75	tension	151	flexure
_8098	1.75	tension	200	flexure

*after failure initiation, it was observed that the crack was propagated via interlaminar shear

V. Failure Analysis

One of the most important parameters to predict in a study of this type is the ultimate loading conditions. This in essence dictates the choice of a failure criterion. The maximum stress theory will be employed in this report because of its simplicity in application and execution. Other popular failure criteria, such as the Tsai-Wu criterion were deemed inappropriate due to the limiting assumptions made in accordance with beam theory.

Maximum stress criterion states that the material will fail when any component of stress exceeds the corresponding material strength. In general, the above statement may be written in equation form as

$$\sigma_i \geq X_i^T \qquad (\sigma_i > 0) \qquad i = 1\text{-}3 \tag{14}$$

$$|\sigma_i| \geq X_i^C \qquad (\sigma_i < 0) \qquad i = 1\text{-}3 \tag{15}$$

$$|\sigma_i| \geq S_i \qquad i = 4\text{-}6 \tag{16}$$

where X_i^T = ultimate tensile strength

X_i^C = ultimate compressive strength

S_i = ultimate shear strength

These equations simplify to those listed below after employing the local coordinate nomenclature for the

"joggle-lap" joint.

$$\sigma_u \geq X^T \qquad (\sigma u>0) \tag{17}$$

$$|\sigma_u| \geq X^C \qquad (\sigma u<0) \tag{18}$$

$$|\sigma_{us}| \geq S_i \tag{19}$$

Applying this failure criterion to the model, it was found that the bottom fiber tensile stresses (see Figure 35) predicted the ultimate loading of the joint within experimental error. Thus the maximum flexural stress was utilized to predict failure.

All failures occurring as a result of tensile loading were initiated along the bottom surface of SEG3. Crack initiation was observed to be of the net tension mode, while propagation appeared to be due to "interlaminar shear". There was a general consistency among the initiation and propagation of the crack for all tension tests.

It was thought at one time that the curved sections of the joint (SEG2, SEG3) were either fiber deficient or highly anisotropic yielding a potential low strength area. However, a photomicrograph of this cross-sectional area clearly shows no such tendencies. (See Plate 7)

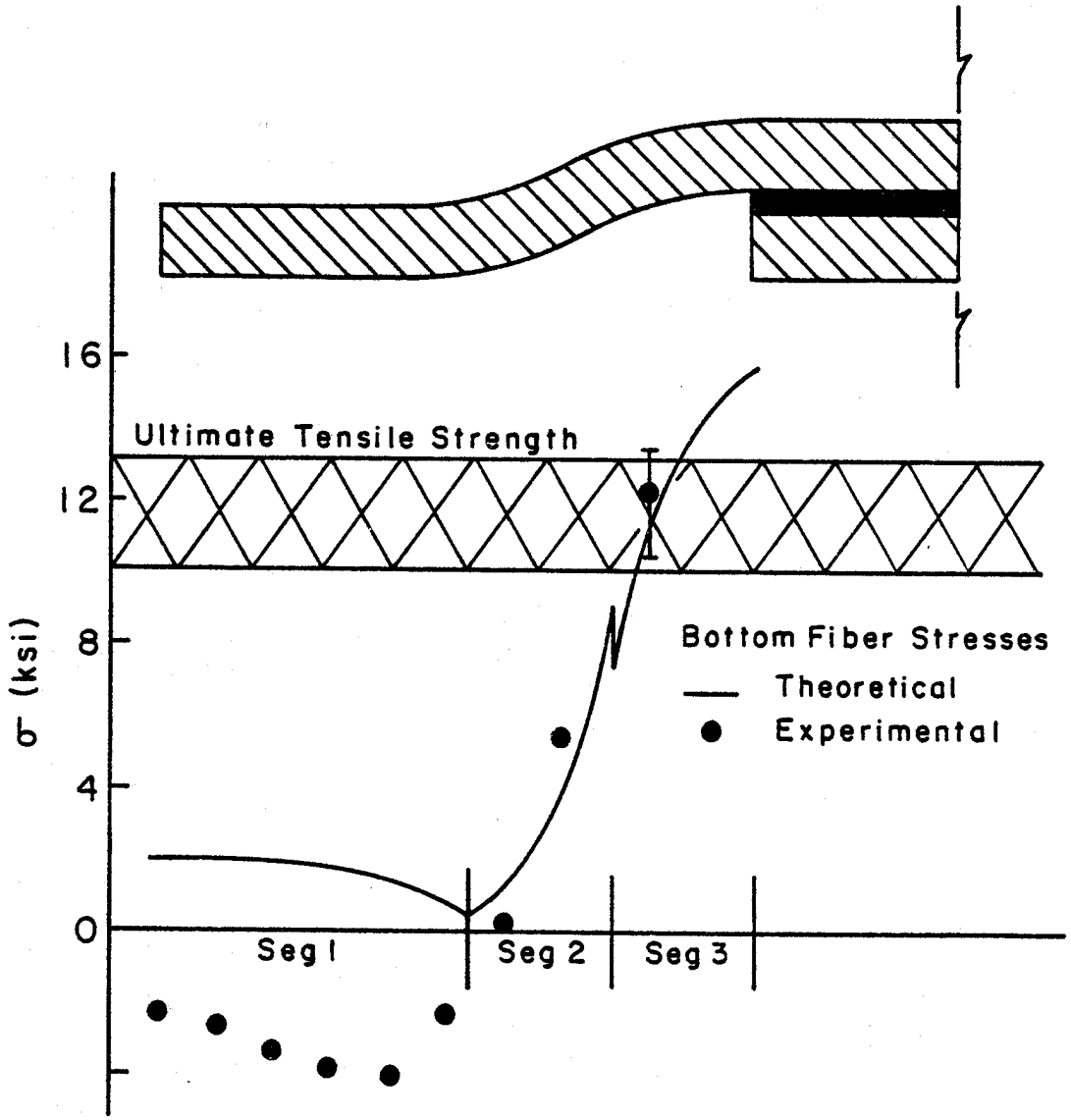

FIGURE 35: BOTTOM FIBER STRESSES AT THE FAILURE LOAD

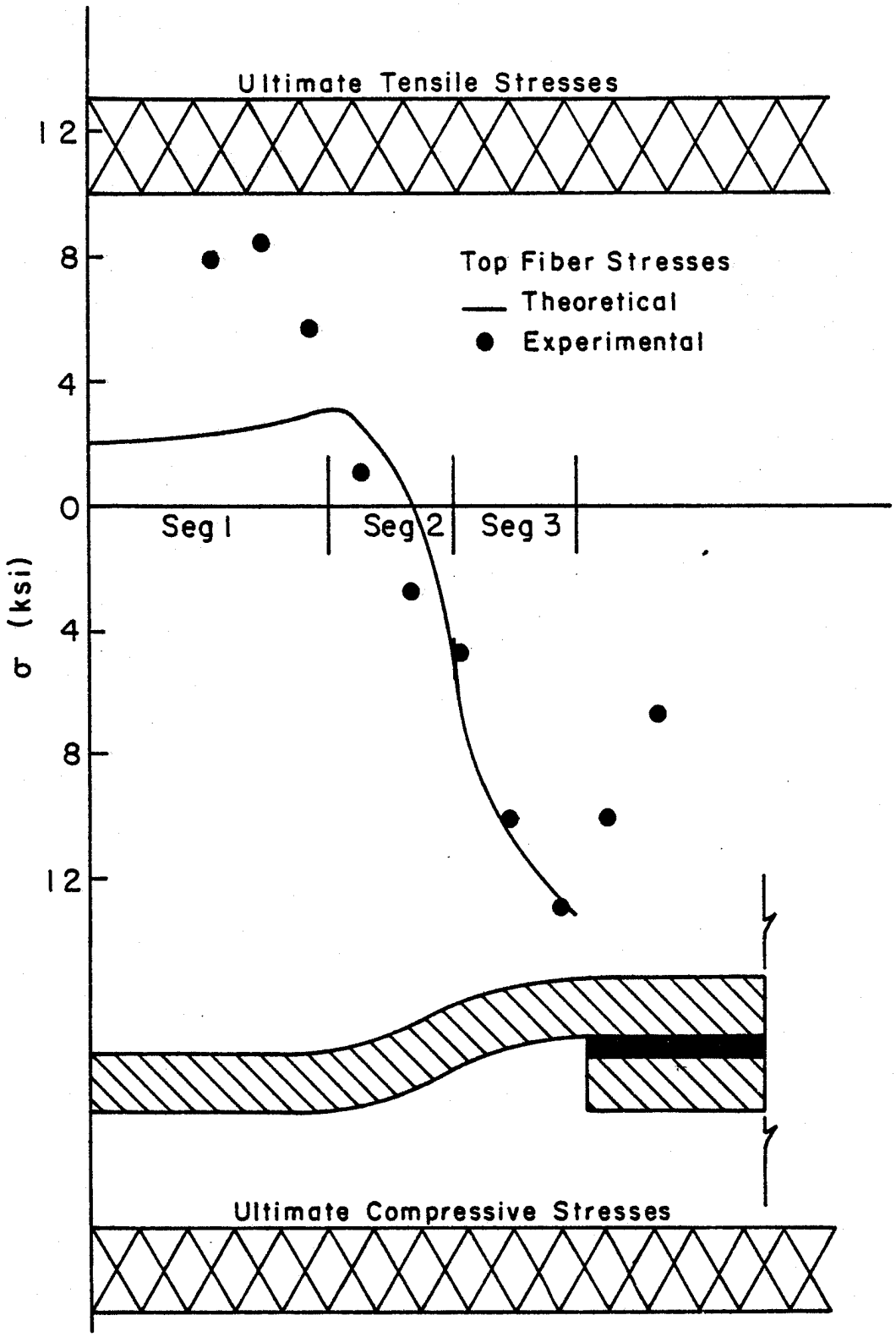

FIGURE 36: TOP FIBER STRESSES AT THE FAILURE LOAD

VI. Conclusions

The response of the "joggle-lap" joint was investigated for both tensile and bending loads in this report. It was found that experimental data correlated rather well to the values of stress predicted by the analytical model. The results of the bending study were not as favorable, in that experimental verification proved to be more difficult.

A parametric study was undertaken for the "joggle-lap" joint subject to tensile loads in an effort to isolate the crucial design parameters. In Figures 37 through 40 a normalized stress value is plotted against one of four parameters - adherent thickness, inside radius, contact area, and load. From these design curves the following conclusions are inferred.

- If weight saving requirements are not stringent, the effect of increasing adherent thickness drastically reduces maximum flexural adherent stress.

- Increasing the radius of curvature has a negligible effect on reducing maximum adherent stress due to a trade-off between mechanisms.

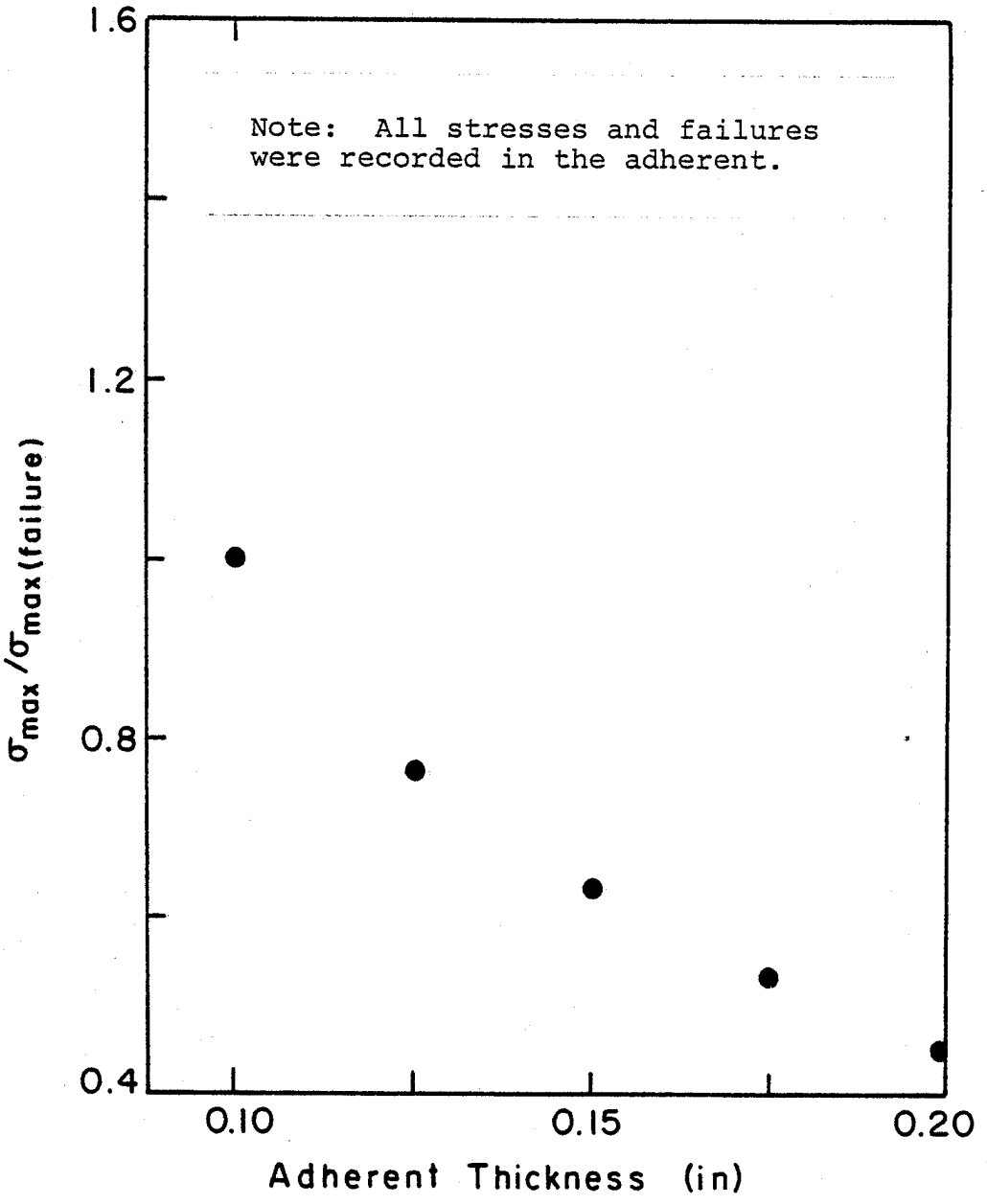

FIGURE 37: EFFECTS OF ADHERENT THICKNESS ON JOINT STRENGTH

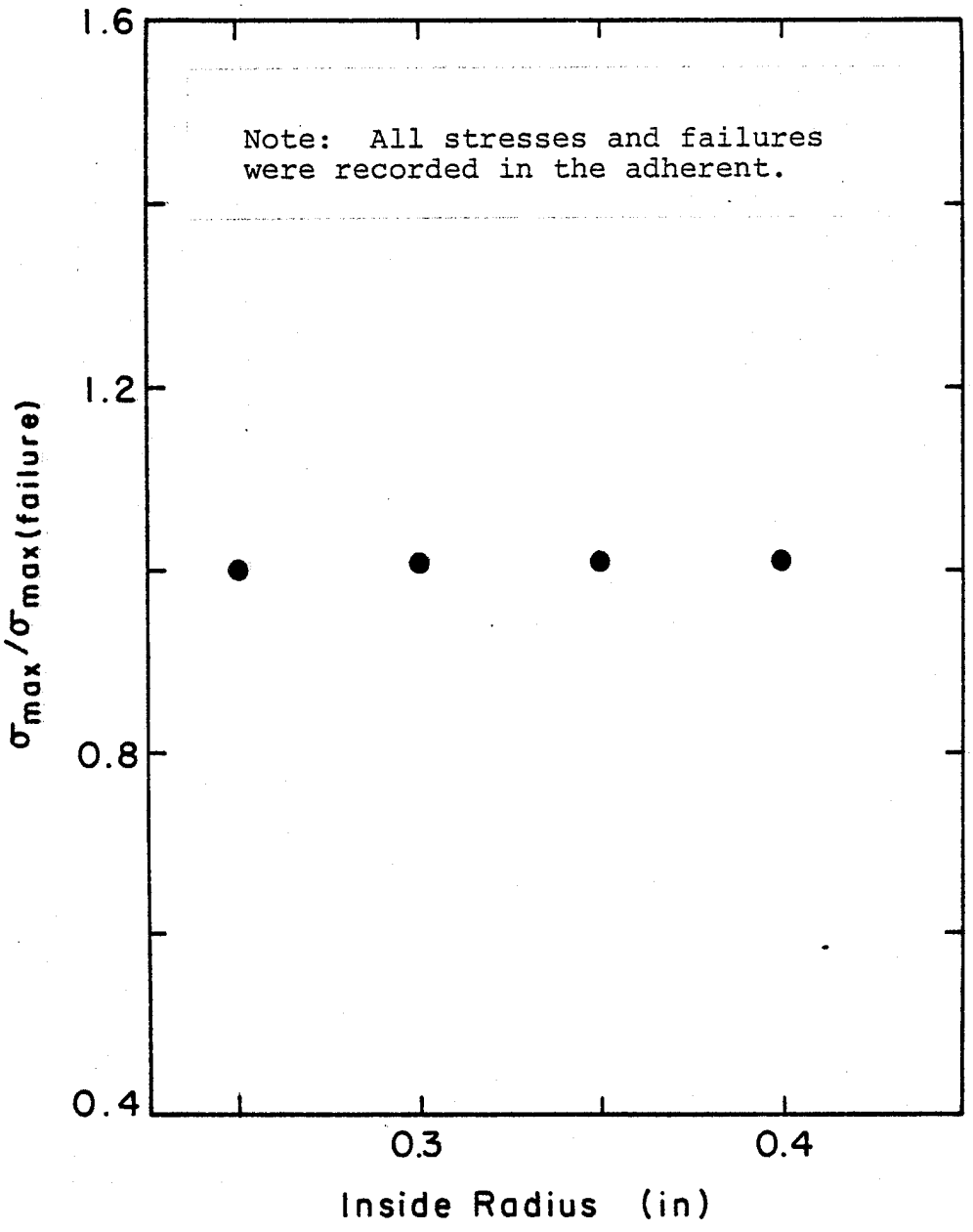

FIGURE 38: EFFECTS OF INSIDE RADIUS OF JOINT STRENGTH

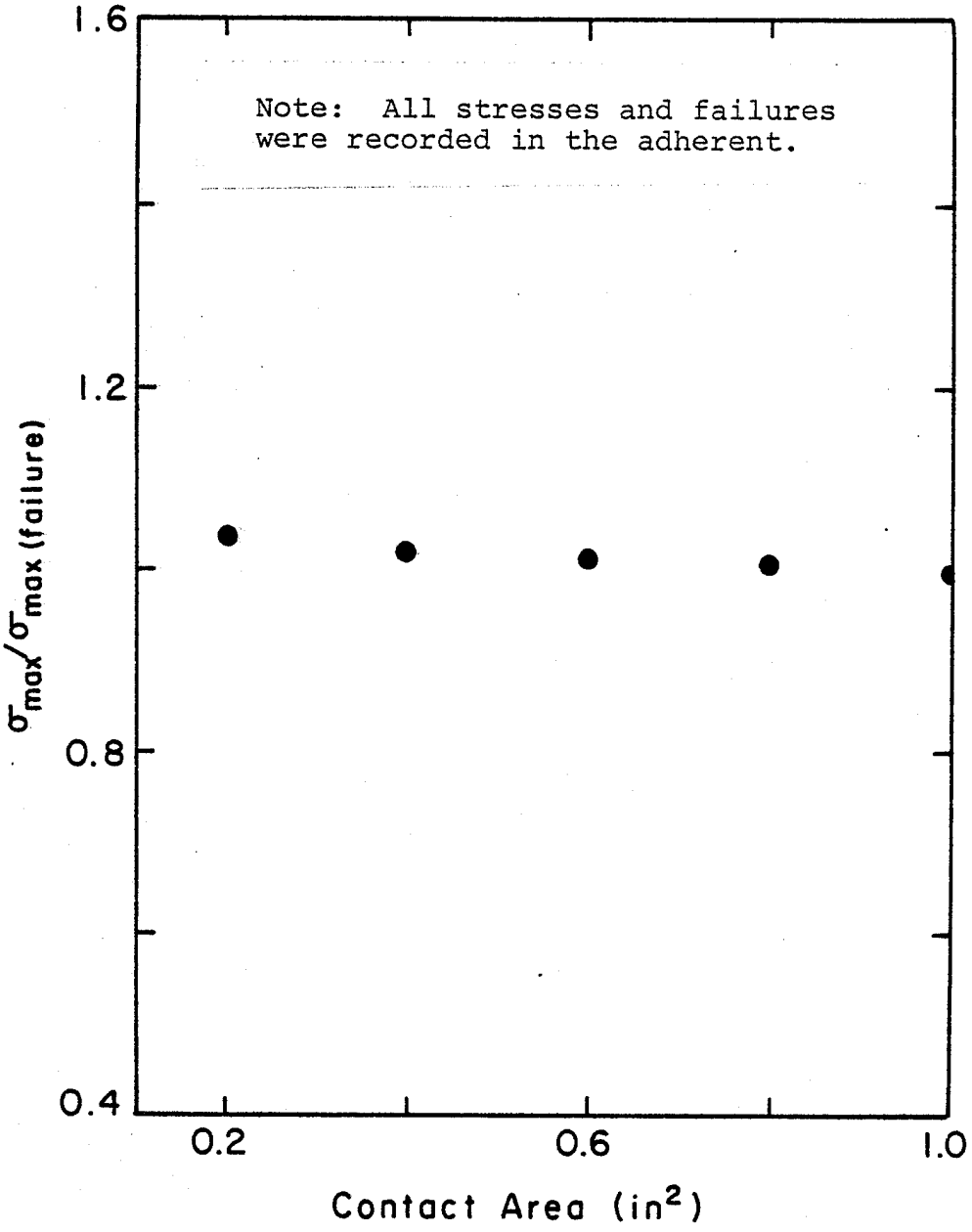

FIGURE 39: EFFECTS OF CONTACT AREA ON JOINT STRENGTH

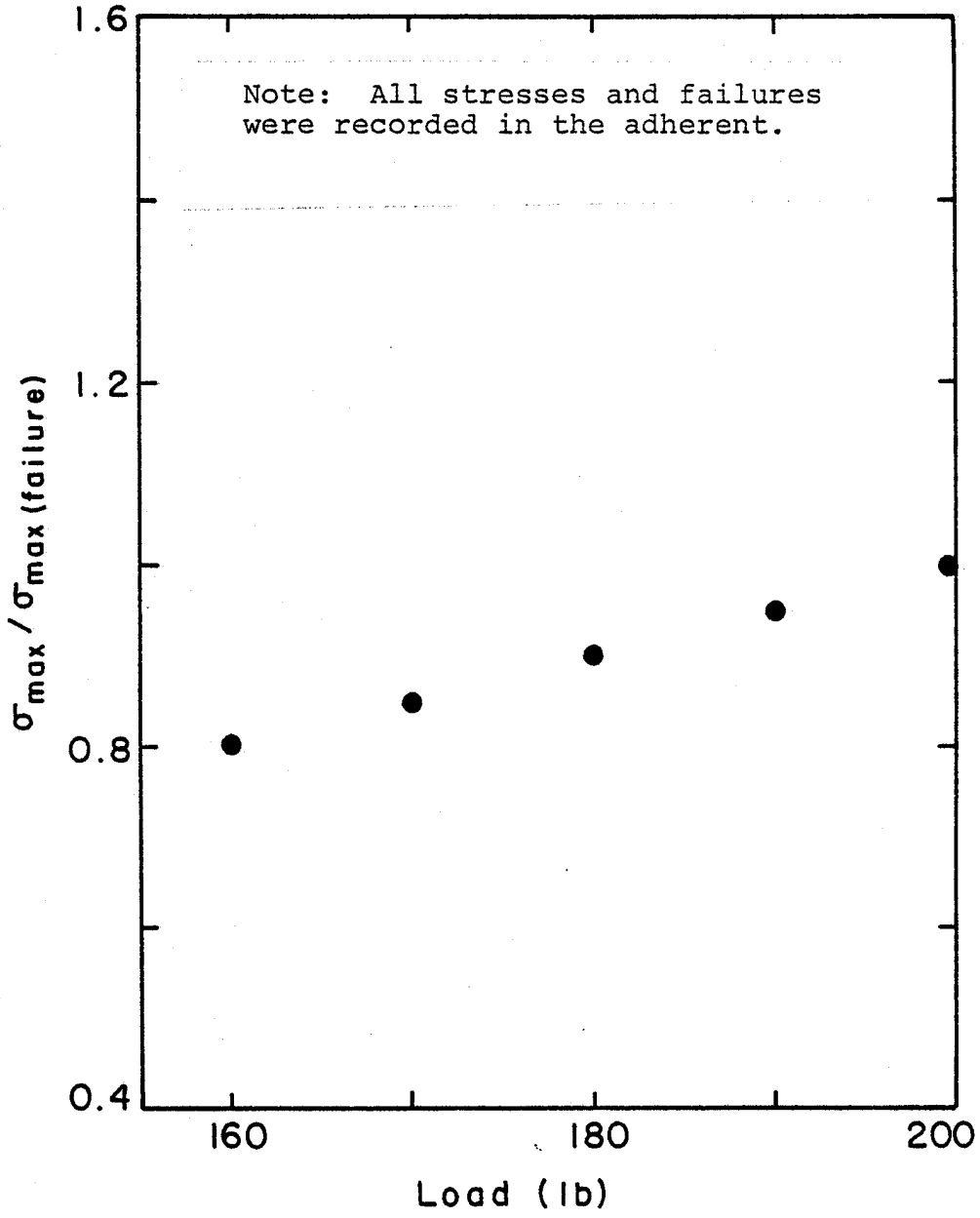

FIGURE 40: EFFECT OF LOAD ON JOINT STRENGTH

- Neglecting local stress concentrations, the effect of reducing the overlap length does not increase adherent stress significantly.

- In the region of the failure load, the maximum adherent stress increases linearly with load.

An important parameter in joint design is that of joint efficiency. This parameter is defined to be the ratio of ultimate joint load divided by the ultimate load carried by the material if the joint were not present. The joint efficiency of the "joggle-lap" joint in tension is calculated to be 0.153.

The adhesive system employed in this report proved to be quite adequate from a structural point of view. For the given overlap length of 1 in (2.54 cm) there were no recorded failures in the adhesive layer. Failure loads were predicted using the maximum flexural stress as the limiting criterion.

This report would be incomplete if it did not offer several suggestions for future work as an outgrowth of this study. An obvious limitation to the work reported herein is the inability to extensively verify the analytical model by experimental testing of various joint geometries.

Further development in this area would greatly increase the reliability of the computer model.

More detailed work needs to be completed in the response of the "joggle-lap" joint to bending loads. This report included only a cursory investigation of bending behavior as a means of identifying the underlying problems associated with the experimental verification of theory.

It is felt that this report will provide a fundamental basis for future research concerning the "joggle-lap" joint.

VII. Acknowledgements

The authors wish to thank David W. Adkins and Joseph J. Quigley, graduate students at the University of Delaware, for their expertise and guidance throughout this research effort. Also, we wish to express our appreciation to Dr. Terry V. Baughn and Bill Englehart of International Harvester for their vested interest in the program and for supplying all of the test specimens. Special thanks are also directed to Larry Carapellotti and his staff at Goodyear Adhesives for their assistance in bonding the experimental specimens.

VIII. List of References

[1] R. Byron Pipes, "Damage Repair Technology for Composite Materials", NASA Grant No. 1304; Semi-annual Progress Report, April 1977

[2] David W. Adkins, "Damage Repair Technology for Composite Materials", NASA Grant No. 1304; Semi-annual Progress Report, May 1978

[3] L. Marker and B. Ford, "Rheology and Molding Characteristics of Glass Fiber Reinforced Sheet Molding Compounds", 32nd Annual Technical Conference, SPI, Inc., 1977

[4] J. W. Gilespie, Jr., "Evaluation of the Embedded Spar Composite Design Concept", Master's Thesis, University of Delaware, 1978

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X. Appendices

Appendix A

Derivation of the Governing Equations
for a Curved Beam

Consider the curved beam element shown in Figure 41.

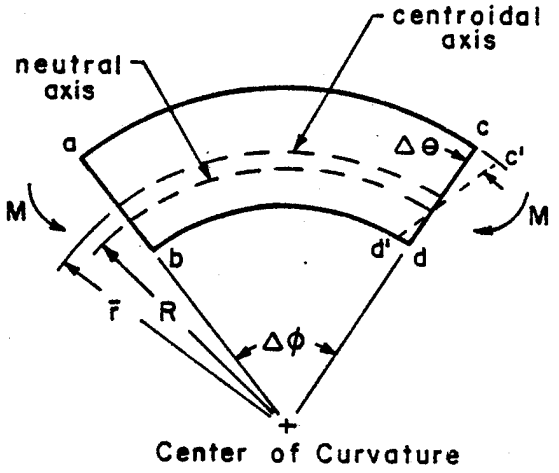

FIGURE 41: CURVED BEAM ELEMENT

The analysis begins by seeking an expression for the strain distribution perpendicular to the neutral axis. Assume that the curved beam, with an initial radius of curvature R, undergoes a small elastic deformation due to the applied moment. (It is important to note that the neutral axis of bending for a curved beam does not necessarily coincide with the centroidal axis of the beam.) Under the action of this moment it becomes apparent that segment cd rotates about the neutral axis

to a new position c'd'. It is assumed here, as in classical beam analysis, that plane sections remain plane. It is readily seen that while the deformation of the beam varies linearly with the distance from the neutral axis, the strains do not. The reason is that the original length of all the fibers prior to the application of the moment are not constant.

Thus the following relation for the strain distribution is written below.

$$\varepsilon = \frac{e_\ell}{\Delta_L} = \frac{-y\Delta\theta}{(R-y)\Delta\phi} \tag{20}$$

where e_ℓ = elongation

y = radial coordinate (positive radially inward)

$\Delta\theta$ = angle of deformation

$\Delta\phi$ = angle subtended by curved beam

The above equation shows the strain to vary hyperbolically across the section. Using the plane stress constitutive relation, Eq. (20) becomes

$$\sigma = \frac{-Ey\ \Delta\theta}{(R-y)\Delta\phi} \tag{21}$$

Now it is appropriate to derive the formulas for flexural stress. First assume that the portion of the beam is in equilibrium. Following directly one may write the equations of equilibrium for an arbitrary section.

$$\Sigma F_{axial} = 0$$

$$\int_A \sigma da = 0 \tag{22}$$

Making the appropriate substitutions for the stress Eq. (22) becomes

$$\int_A \frac{-Ey\Delta\theta da}{(R-y)\Delta\phi} = 0 \tag{23}$$

Assuming E, $\Delta\phi$, and $\Delta\theta$ to be constants the integral is simplified as shown in Eq. (24).

$$\int_A \frac{yda}{(R-y)} = 0 \tag{24}$$

It is possible to solve Eq. (24) for the radius of curvature and thus locate the neutral surface; however, it will suffice to let Eq. (24) stand as is for now.

Referring to Figure 42 and summing moments about the

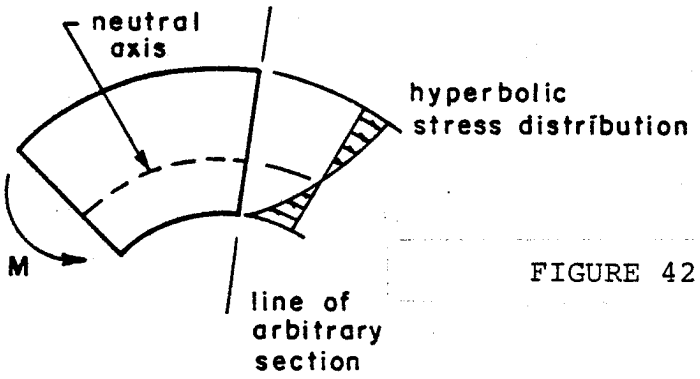

FIGURE 42

neutral axis, one finds that the stress distribution must also satisfy the equation below.

$$M = -\int_A \sigma \, y \, da \tag{25}$$

making the appropriate substitutions, Eq. (25) becomes

$$M = -\int_A E\left(\frac{-y\Delta\theta}{(R-y)\Delta\phi}\right) y \, da \tag{26}$$

$$M = \frac{E\Delta\theta}{\Delta\phi} \int_A \frac{y^2}{(R-y)} da \tag{27}$$

Notice the algebraic relation that permits the substitution of an equivalent expression into Eq. (27).

$$\frac{y^2}{R-y} = \frac{Ry}{R-y} - y \tag{28}$$

Eq. (27) now becomes

$$M = \frac{E\Delta\theta}{\Delta\phi}\left[\int_A \frac{Ry \, da}{R-y} - \int_A y \, da\right] \tag{29}$$

and from the result of Eq. (24)

$$M = \frac{E\Delta\theta}{\Delta\phi} \left(R(0) - a\bar{u}\right) \tag{30}$$

where a = area

$\bar{u}$ = distance between the neutral and centroidal axes

Rearranging Eq. (30) yields

$$\frac{\Delta\theta}{\Delta\phi} = \frac{-M}{Ea\bar{u}} \tag{31}$$

Comparing this equation with the well-known deflection equation for straight beams, it is apparent that

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \tag{32}$$

the left hand side of Eq. (31) is not yet suitable. The ultimate goal of such an analysis is to seek an equation that relates the deflection of the neutral axis to the position along the neutral axis.

Consider Figure 43 shown below.

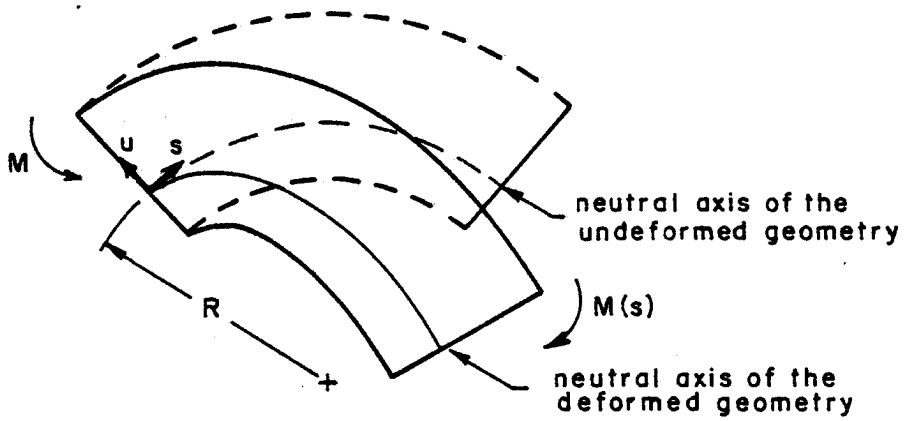

FIGURE 43: CURVED BEAM ELEMENT SUBJECT TO DEFLECTION

The beam is deflected as shown to illustrate the most general case of a non-constant moment. That is, the moment is a function of position. Now the deflection can be measured as the deviation between the undeformed neutral surface and the deformed neutral surface. For convenience just the neutral axis and appropriate parameters are drawn in Figure 44.

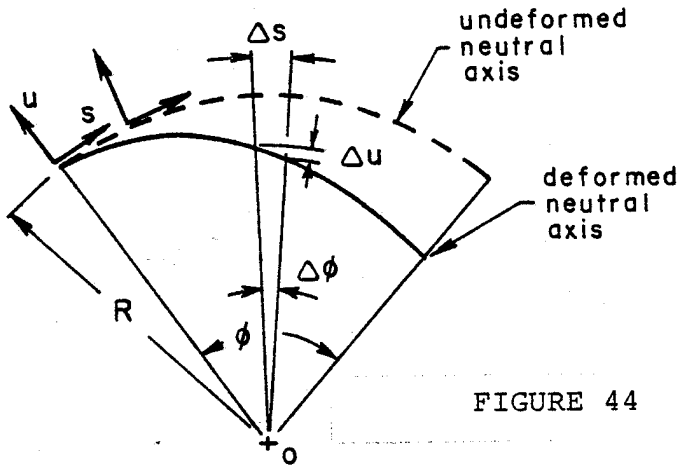

FIGURE 44

A coordinate system u,s is defined and shown in the figure where s traverses tangentially to the undeformed neutral axis and u is defined to be perpendicular to that axis.

Enlarging the area of interest and focusing on the triangle of Figure 45, one finds that

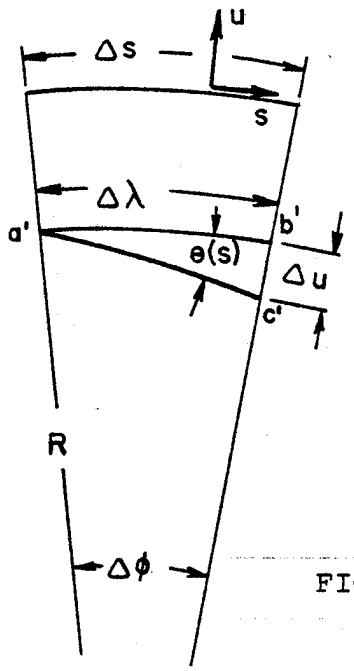

FIGURE 45

$$\theta(s) = \frac{\Delta u}{\Delta \lambda}$$

Realizing that $\tan \alpha \tilde{=} \alpha$ for small α, it follows that

$$\Delta \lambda = \frac{(R+u)\Delta S}{R}$$

and thus

$$\frac{\theta(s)}{} = \frac{R\Delta u}{(R+u)\Delta s}$$

which may be written as

$$\frac{\Delta u}{\Delta s} = \frac{(R+u)\,\theta(s)}{R} \tag{33}$$

Finally in the limit as $\Delta s \to 0$: Eq. (33) becomes

$$\lim_{\Delta s \to 0} \frac{\Delta u}{\Delta s} = \frac{(R+u)\theta(s)}{R} = \frac{du}{ds} \tag{34}$$

From Eq. (31), several simplifications can be made with the proper substitutions.

$$\frac{\Delta \theta}{\Delta \theta} = \frac{-M}{Ea\bar{u}}$$

where $$\Delta \phi = \frac{\Delta s}{R}$$

$$\lim_{\Delta s \to 0} \frac{\Delta \theta}{\Delta s} = \frac{-M}{REa\bar{u}} = \frac{d\theta}{ds} \tag{35}$$

Differentiating Eq. (34) with respect to s yields

$$\frac{d^2u}{ds^2} = \frac{(R+u)}{R}\frac{d\theta}{ds} \tag{36}$$

and substituting Eq. (35) into Eq. (36) yields the final results - a second order differential equation relating deflection to position in terms of the applied moment.

$$\frac{d^2u}{ds^2} = \frac{(R+u)M}{R^2E\bar{a}\bar{u}} \tag{37}$$

Appendix B

Beam Bending Model of the "Joggle-Lap" Joint

SEG1 may be modeled as a straight beam shown in Figure 46.

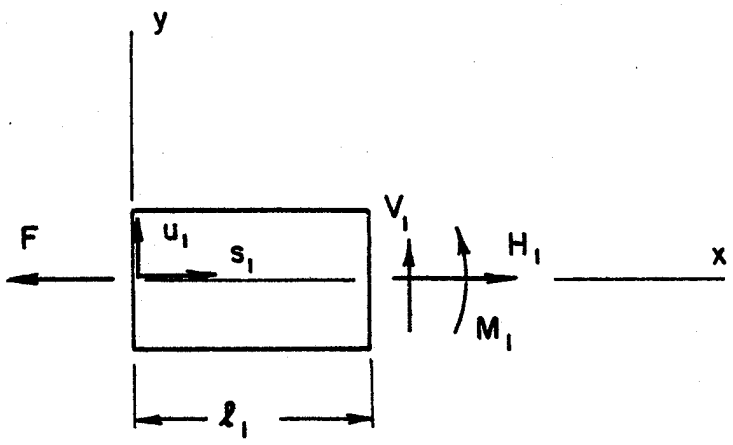

FIGURE 46: SEG1 MODELED AS A STRAIGHT BEAM

In general, the moment experienced by any segment originates from two sources: eccentricity from geometry and eccentricity due to deflection. The preceding statement may be written algebraically as follows.

$$M = F(e_{geom} + e_{defl}) \tag{38}$$

where M = moment

F = applied force

e = eccentricity

It is readily seen that $e_{geom} = 0$ for SEG1. Writing Eq. (38) in the local coordinate system, the moment experienced by

this segment reduces to

$$M = Fu_1 \tag{39}$$

where

u_1 = deflection in the local coordinate system

Substituting Eq. (39) into Eq. (5) yields

$$\frac{d^2u_1}{ds_1^2} - \frac{Fu_1}{EI} = 0 \tag{40}$$

The corresponding boundary conditions are expressed below

$$u_1(0) = 0$$
$$u_1(\ell_1) = u_o$$

where u_o is yet undetermined.

The solution of Eq. (40) is of standard form and known to be

$$u_1 = C_1 \sinh\sqrt{F/EI} \cdot s_1 + C_2 \cosh\sqrt{F/EI} \cdot s_1 \tag{41}$$

Applying the boundary conditions to Eq. (41) determines the constants C_1 and C_2 to be

$$C_2 = 0$$
$$C_1 = u_o \Big/ \sinh\sqrt{F/EI} \cdot \ell_1$$

and thus

$$u_1 = u_o \frac{\sinh \sqrt{F/EI} \cdot s_1}{\sinh \sqrt{F/EI} \cdot \ell_1} \qquad 0<s_1<\ell_1 \tag{42}$$

where u_o is necessarily negative to correspond with the physical system. In other words, for a given tensile load it is expected that SEG1 will deflect downward. (Figure 6). Also

$$\frac{du_1}{ds_1}(\ell_1) = u_o \sqrt{F/EI} \frac{\cosh \sqrt{F/EI}\ \ell_1}{\sinh \sqrt{F/EI}\ \ell_1}$$

It should be noted that the deflection as given by Eq. (42) is not known explicitly in terms of the given parameters. U_o is still unknown and it will be shown later how this value may be determined uniquely.

SEG2 is modeled as a curved beam and shown in Figure 47. The local coordinate system is a curvilinear coordinate system with the s_2 axis traversing the neutral axis as shown. Positive deflections are measured normal to the undeformed neutral axis in the direction of u_2.

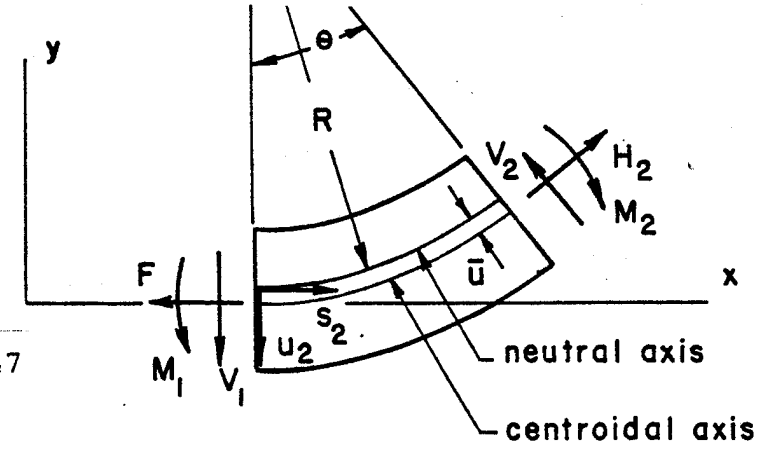

FIGURE 47

From the derivation of the general case for a curved beam in pure bending (see Appendix A), the governing equation for the deflection is

$$\frac{d^2u_2}{ds_2^2} = \frac{(R + u_2)M}{R^2Ea\bar{u}} \tag{43}$$

where

s_2 = arc length

u_2 = deflection normal to neutral axis

M = moment

R = radius of curvature

E = modulus of elasticity

a = cross sectional area

$\bar{u}$ = distance between neutral axis and centroidal axis and its value is necessarily negative

The moment may be written as the product of the applied load and the eccentricity, where the eccentricity in this case consists of both geometry and deflection considerations.

At this point, it is appropriate to introduce the notion of extensional effects. It is realized that with the given loading conditions, the "joggle-joint" will undergo deflections parallel to the neutral axis as well. This fact would be of little concern if all beam segments of the joint configuration had their neutral axis aligned with the loading axis. If this were the case, the longitudinal displacement would not affect the eccentricity.

However, it is evident that the extensional strains in the curved beam segments give rise to an added component of eccentricity defined to be e_{ext}. To calculate the value of e_{ext}, one merely applies the criterion of force equilibrium to SEG2 (Figure 47) in the local coordinate system.

$$\Sigma F_{u_2} = 0 \qquad H_2\cos\theta + V_2\sin\theta = F$$

$$\Sigma F_{s_2} = 0 \qquad H_2\sin\theta = V_2\cos\theta$$

thus $H_2 = F\cos\theta$

where θ = angle subtended by SEG2

Employing the constitutive relationship

$$\sigma = E\varepsilon$$

where σ = stress

E = modulus of elasticity

ε = strain

and considering only the y (global coordinate) component of the extension we thus arrive with the expression for e_{ext}.

$$e_{ext} = \frac{Fs_2\cos(s_2/R)\sin(s_2/R)}{aE} \tag{44}$$

Eq. (44) must be added to the other terms which comprise the eccentricity due to deflection.

Therefore Eq. (43) becomes

$$\frac{d^2u_2}{ds_2^2} = \frac{-(R-u_2)F}{R^2Ea\bar{u}} [e_{geom} + e_{defl} + e_{ext}] \qquad (45)$$

where $e_{geom} = R(1-\cos(\frac{s_2}{R}) + \bar{u})$

$e_{defl} = u_2\cos(\frac{s_2}{R})$

$e_{ext} = Fs_2\cos\frac{(s_2/R)\sin(s_2/R)}{aE}$

Initial conditions for SEG2 are found by matching deflection and slope at the 1-2 interface.

$$u_2(0) = u_o$$

$$\frac{du_2}{ds_2}(0) = u_o\sqrt{F/EI}\ \frac{\cosh\sqrt{F/EI}\ \ell_1}{\sinh\sqrt{F/EI}\ \ell_1}$$

Using a numerical integration routine to solve Eq. (45) the deflection u_2 may be marched out as a function of arc length s_2. A Runge-Kutta method based on Verners fifth and sixth order pair of formulas was used. An explanation of the integration routine DVERK may be referenced in Appendix C.

Figure 48 shows SEG3 modeled as a curved beam. From Eq. (37) the governing differential equation for a curved beam in pure bending is

$$\frac{d^2u_3}{ds_3^2} = \frac{(R+u_3)}{R^2Ea\bar{u}} M \qquad (46)$$

where $M = F(e_{geom} + e_{defl} + e_{ext})$

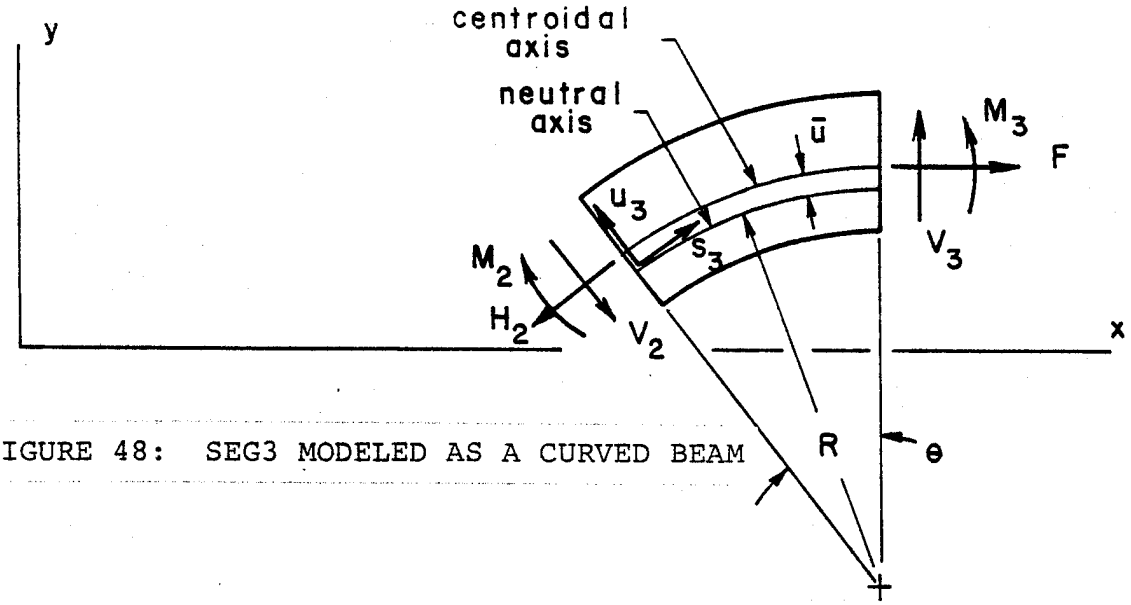

FIGURE 48: SEG3 MODELED AS A CURVED BEAM

Through geometric considerations e_{geom} can be shown to be

$$e_{geom} = \bar{u} + R(1-\cos\theta) - 2\bar{u}\cos\theta + R\sin(\tfrac{\pi}{2} - \theta + {}^{s}3/R) - \cos\theta \qquad (47)$$

where θ = angle subtended by SEG3

$\bar{u}$ = distance between centroidal and neutral axes

R = radius of curvature

s_3 = arc length along neutral surface of SEG3

Also $e_{defl} = M \cos(\pi/2 - \theta + s3/R)$ (48)

From a similar argument developed earlier it may be shown that e_{ext} for SEG3 is given by

$$e_{ext} = \frac{Fs_3\cos(\theta - s_3/R)\ \sin\ (\theta - s_3/R)}{aE} \qquad (49)$$

Thus Eq. (46) becomes

$$\frac{d^2u_3}{ds_3^2} = \frac{-(R+u)}{R^2 E a \bar{y}} F \left(e_{geom} + e_{defl} + e_{ext}\right) \tag{50}$$

where e_{geom}, e_{defl}, and e_{ext}, are given by Eqs. (47), (48), and (49) respectively. Matching boundary conditions at the 2-3 interface provides initial conditions to Eq. (50) which may be integrated numerically as before.

SEG4 is analyzed as a multi-layered beam and shown in Figure 49. Treating this segment to be composed of three linear elastic beam elements, the governing differential equation follows from Eq. (5) with a slight modification.

$$\frac{d^2u_4}{ds_4^2} = \frac{M}{\sum_{i=1}^{3} E_i I_i} \tag{51}$$

where I_i = moment of intertia of the ith section about the neutral axis

E_i = modulus of elasticity of the ith element

$\sum_{i=1}^{3} E_i I_i$ is referred to as an effective flexural stiffness and is merely a constant. The moment is defined in the usual manner as

$$M = F(e_{geom} + e_{defl})$$

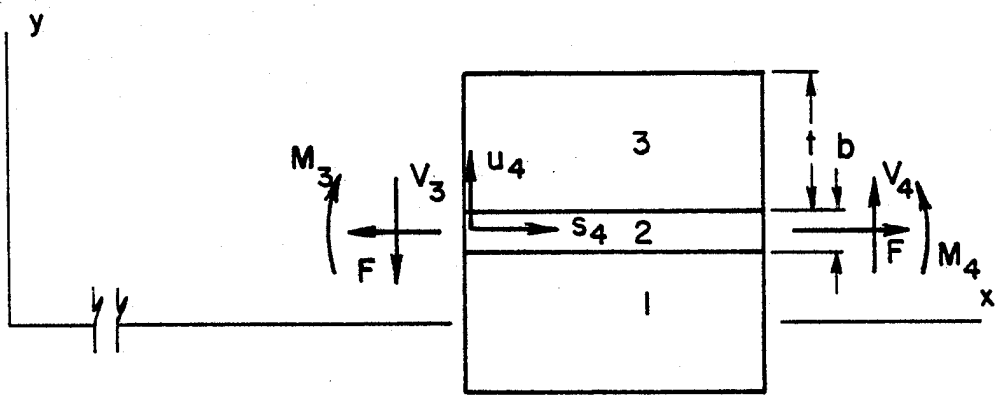

FIGURE 49: SEG4 MODELED AS A LAYERED BEAM

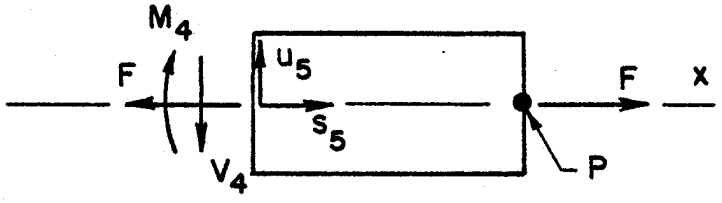

FIGURE 50: SEG5 MODELED AS A STRAIGHT BEAM

where $e_{geom} = .5(t+b)$

$e_{defl} = u_4$

Thus Eq. (51) becomes

$$\frac{d^2u_4}{ds_4^2} = \frac{F\ (.5(t+b) + u_4)}{\sum_{i=1}^{3} E_i I_i} \tag{52}$$

Initial conditions are found by equating the deflection and slope at the 3-4 interface. Following in the usual manner, Eq. (52) is integrated to obtain an expression for the deflection of SEG4 as a function of arc length in the local coordinate system.

Finally SEG5 is shown in Figure 50 modeled as a straight beam member. The governing differential equation is the same as Eq. (5)

$$\frac{d^2u_5}{ds_5^2} = \frac{M}{EI} \tag{53}$$

where $M = Fu_5$

and the initial conditions are obtained by matching the deflection and slope at the 4-5 interface. Upon integration of Eq. (53) the deflection SEG5 will be a known function of the abscissa s_5 of the local coordinate system. Therefore the deflection and slope at point P of Figure 50 are also known. But it should be apparent that the values of the deflection

and slope at this point must be zero or at least within certain tolerance limits. This in fact is the final boundary condition to the problem that is needed to uniquely determine the value of u_o which was previously assumed to be arbitrary. Thus, through an iterative process, a correct value of u_o may be calculated by assuring that the deflection and slope of point P of SEG5 is sufficiently[3] close to zero. To avoid confusion, it should be noted that by specifying zero deflection at point P we will force the slope to zero by the nature of the deflection function of SEG5. So in fact this is a well-posed problem, whereby we specify only enough boundary conditions as there are unknowns. The process for correctly determining u_o is shown schematically in Figure 51.

---

[3] it was found that reliable results were obtained by using the tolerance limits listed here.

$$|\text{deflection (P)}| < .00001$$
$$|\text{slope (P)}| < .00005$$

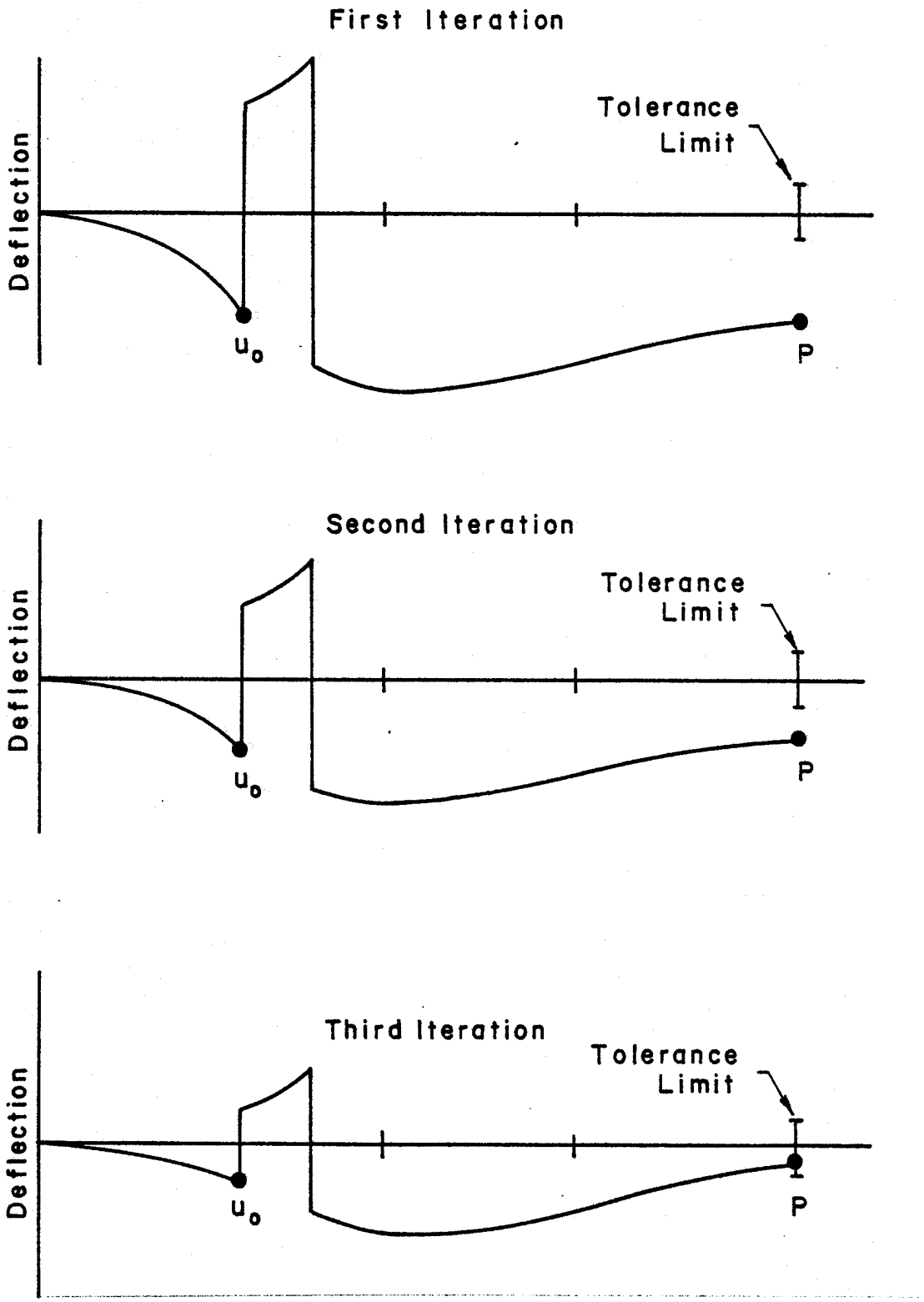

FIGURE 51: ITERATIVE PROCESS FOR DETERMINING u_o

Appendix C

Computer Programs

a. JOGGLE

To facilitate ease of calculation, a computer routine identified as JOGGLE was developed and may be referenced below. Essentially this program calculates a correct value of u_o and proceeds to determine a solution for the deflection while calculating stress profiles along the joint configuration. These stress profiles are linear in the straight beam members (SEG1, SEG5) and hyperbolic in the curved beams (SEG2, SEG3).

```
C-                  J   00000   GGGGG   GGGGG   L       EEEEE          00000003
C-                  J   0   0   G       G       L       E              00000004
C-                  J   0   0   G  GG   G  GG   L       EEE            00000005
C-              J   J   0   0   G   G   G   G   L       E              00000006
C-              JJJJJ   00000   GGGGG   GGGGG   LLLLL   EEEEE          00000007
C-                                                                     00000008
C-                                                                     00000009
C-                                                                     00000010
C-                                                                     00000011
C-                                                                     00000015
C-                                                                     00000016
C-                  ANALYTICAL BEAM BENDING MODEL                      00000020
C-                       FOR A JOGGLE LAP JOINT                        00000021
C-                                                                     00000024
C-                                                                     00000025
C-                DEVELOPED BY:  RICHARD C. GIVLER                     00000026
C-                     UNIVERSITY OF DELAWARE                          00000027
C-                       SEPT 78 - JAN 79                              00000028
C-                                                                     00000040
C-                                                                     00000041
C-                                                                     00000042
C-                                                                     00000043
$SET AUTOBIND                                                          00000100
$RESET FREE                                                            00000110
FILE  6(KIND=REMOTE,MAXRECSIZE=22)                                     00000120
      DIMENSION PROD(3), ERR(1), T(2),  EI(3),  YPRIME(2)              00000150
      COMMON R, PLOAD, ESMC, WIDTH, THICK, YBAR, THETA, EADH, PI       00000175
     -, BOND, J, TORC, TORCS, TRAC, TRACE, SHEAR                       00000176
C-                                                                     00000178
C-                                                                     00000180
C-----PARAMETERS AND NOMENCLATURE                                      00000200
C-                                                                     00000210
C-                                                                     00000220
C-----MATERIAL THICKNESS                                               00000300
      THICK=.1                                                         00000400
C-----LONGITUDINAL MODULUS OF ELASTICITY OF SMC IN PSI                 00000500
      ESMC=2.1E+06                                                     00000600
C-----MODULUS OF ELASTICITY OF ADHESIVE IN PSI                         00000700
      EADH=1.0E+05                                                     00000800
C-----LOAD IN LBS                                                      00000900
      PLOAD=200.                                                       00001000
C-----INSIDE RADIUS IN INCHES                                          00001100
      RADI=2.5*THICK                                                   00001200
C-----OUTSIDE RADIUS IN INCHES                                         00001300
      RADO=3.5*THICK                                                   00001400
C-----BONDING THICKNESS IN INCHES                                      00001500
      BOND=.03                                                         00001600
C-----CONTACT WIDTH IN INCHES                                          00001700
      CONTA=1.0                                                        00001800
C-----SPECIMEN WIDTH                                                   00001900
      WIDTH=1.0                                                        00002000
C-----LENGTH OF SEGMENT 1                                              00002100
      SEGA=3.5                                                         00002200
C-----LENGTH OF SEGMENT 5                                              00002300
      SEGB=4.0                                                         00002400
C-----LEFT INTERVAL LIMIT FOR ITERATION                                00002420
      AINT=-.04                                                        00002430
C-----RIGHT INTERVAL LIMIT FOR ITERATION                               00002435
      BINT=+.01                                                        00002440
C-----TOLERANCE LIMIT ON INITIAL DISPLACEMENT                          00002445
      ERR(1)=.00001                                                    00002450
```

```
C-----STEP SIZE FOR NUMERICAL INTEGRATION                                00002451
      STEP=50.                                                           00002452
      PI=3.141592654                                                     00002475
C-----TRACING CONSTANTS                                                  00002500
      TRAC=1.0                                                           00002501
      TRACE=1.0                                                          00002502
C------                                                                  00002600
C-----NOTE:DEFLECTIONS ARE MEASURED NORMAL TO THE UNDEFORMED             00002700
C-     NEUTRAL AXIS                                                      00002800
C-                                                                       00002900
      YO=(AINT+BINT)/2.                                                  00003000
C-                                                                       00003100
C-----ITERATIVE CALCULATION OF YO TO FORCE ZERO DEFLECTION               00003200
C-----AND SLOPE AT END OF SEG5                                           00003210
C-----YO MUST LIE BETWEEN THE PROPOSED LIMITS OF AINT AND BINT           00003220
C-                                                                       00003400
C-***********************************************************           00003405
C-***********************************************************           00003410
C-***********************************************************           00003420
C NUMERICAL INTEGRATION VIA LIBRARY ROUTINES DVERK AND UERTST            00003425
C-***********************************************************           00003430
C-***********************************************************           00003435
C-***********************************************************           00003440
C-                                                                       00003445
 101  CONTINUE                                                           00003450
      DO 200 X=0.,SEGA+.005,SEGA/STEP                                    00004200
      PROD(3)=SQRT(PLOAD/(ESMC*(WIDTH*THICK**3./12.)))                   00004300
      PROD(1)=YO*(.5*(EXP(PROD(3)*X)-EXP(-PROD(3)*X)))                   00004400
      PROD(2)=.5*(EXP(PROD(3)*SEGA)-EXP(-PROD(3)*SEGA))                  00004500
      T(1)=PROD(1)/PROD(2)                                               00004600
      T(2)=YO*PROD(3)/PROD(2)*(.5*(EXP(PROD(3)*X)+EXP                    00004700
     -(-PROD(3)*X)))                                                     00004800
 200  CONTINUE                                                           00005100
      DIMENSION  C(24), Y(2),  W(2,9)                                    00006000
      EXTERNAL FCN1                                                      00006100
C-----CALCULATION OF THE RADIUS OF CURVATURE FOR CURVED MEMBERS          00006150
      R=WIDTH*THICK/(ALOG(RADO/RADI))                                    00006200
      NW=2                                                               00006300
C-----CALCULATION OF THE ANGLE SUBTENDED BY SEG2 AND SEG3                00006350
      THETA=ARCOS((5.*THICK-BOND)/(6.*THICK))                            00006400
      N=2                                                                00006500
C-----CALCULATION OF THE DISTANCE BETWEEN NEUTRAL AXIS AND               00006550
C-----CENTROIDAL AXIS OF CURVED MEMBERS                                  00006552
      YBAR=RADI+THICK/2.-R                                               00006600
      X=0.0                                                              00006700
      Y(1)=-T(1)                                                         00006800
      Y(2)=-T(2)                                                         00006900
      TOL=.000001                                                        00007000
      IND=1                                                              00007100
      DO 300 Z=0.0,R*THETA+.001,R*THETA/STEP                             00007200
      XEND=FLOAT(Z)                                                      00007300
      CALL DVERK(N,FCN1,X,Y,XEND,TOL,IND,C,NW,W,IER)                     00007400
      IF(IND.LT.0.OR.IER.GT.0) GO TO 20                                  00007500
 300  CONTINUE                                                           00007800
  20  CONTINUE                                                           00007900
      RINT=RNEW                                                          00007950
      EXTERNAL FCN2                                                      00008600
      X=0.0                                                              00008700
      Y(1)=-Y(1)                                                         00008720
      Y(2)=-Y(2)                                                         00008730
      NW=2                                                               00008800
      N=2                                                                00008900
```

```
      IND=1                                                              00009000
      DO 250 M=1,24                                                      00009100
      C(M)=0.0                                                           00009200
  250 CONTINUE                                                           00009300
      DO 350 Z=0.0,R*THETA+.001,R*THETA/STEP                             00009400
      XEND=FLOAT(Z)                                                      00009500
      CALL DVERK(N,FCN2,X,Y,XEND,TOL,IND,C,NW,W,IER)                     00009600
      IF(IND.LT.0.OR.IER.GT.0) GO TO 70                                  00009700
  350 CONTINUE                                                           00010000
   70 CONTINUE                                                           00010100
      EXTERNAL FCN3                                                      00011000
      X=0.0                                                              00011100
      NW=2                                                               00011200
      N=2                                                                00011300
      IND=1                                                              00011400
      DO 290 M=1,24                                                      00011500
      C(M)=0.0                                                           00011600
  290 CONTINUE                                                           00011700
      DO 400 Z=0.0,CONTA+.005,CONTA/STEP                                 00011800
      XEND=FLOAT(Z)                                                      00011900
      CALL DVERK(N,FCN3,X,Y,XEND,TOL,IND,C,NW,W,IER)                     00012000
      IF(IND.LT.0.OR.IER.GT.0) GO TO 80                                  00012100
  400 CONTINUE                                                           00012400
   80 CONTINUE                                                           00012500
      EXTERNAL FCN4                                                      00013400
      X=0.0                                                              00013500
      NW=2                                                               00013600
      N=2                                                                00013700
      IND=1                                                              00013800
      DO 291 M=1,24                                                      00013900
      C(M)=0.0                                                           00014000
  291 CONTINUE                                                           00014100
      DO 246 Z=0.0,SEGB+.005,SEGB/STEP                                   00014200
      XEND=FLOAT(Z)                                                      00014300
      CALL DVERK(N,FCN4,X,Y,XEND,TOL,IND,C,NW,W,IER)                     00014400
      IF(IND.LT.0.OR.IER.GT.0) GO TO 81                                  00014500
  246 CONTINUE                                                           00014800
   81 CONTINUE                                                           00014900
      IF(ABS(Y(1)).LT.ERR(1)) GO TO 88                                   00014961
      IF(Y(1).GT.0) GO TO 100                                            00014962
      IF(Y(1).LT.0) GO TO 89                                             00014963
  100 CONTINUE                                                           00015000
      BINT=YO                                                            00015075
      YO=(AINT+BINT)/2.                                                  00015076
      GO TO 101                                                          00015077
   89 CONTINUE                                                           00015080
      AINT=YO                                                            00015085
      YO=(AINT+BINT)/2.                                                  00015086
      GO TO 101                                                          00015087
   88 CONTINUE                                                           00015099
C-*****************************************************************    00016020
C-*****************************************************************    00016030
C-*****************************************************************    00016040
C-                                                                       00024000
C-----CALCULATION OF DEFLECTION AND SLOPE AS A FUNCTION                  00024100
C-      OF X FOR SEGMENT 1                                               00024200
C-                                                                       00024300
      WRITE(6,500) YO                                                    00024400
  500 FORMAT(//////////,10X,'YO=',F14.11,)                               00024500
      WRITE(6,501)                                                       00024600
  501 FORMAT('----------------------------------------')                 00024700
      WRITE(6,502)                                                       00024800
```

```
 502  FORMAT('----------------------------------------')                C0024900
      WRITE(6,2)                                                        C0025000
    2 FORMAT(/////,' DEFLECTION AND SLOPE FOR SEGMENT 1',//)            C0025100
      WRITE(6,9)                                                        C0025200
    9 FORMAT(10X,'X DISTANCE',5X,'DEFLECTION',7X,'SLOPE',9X,'STRESS'    C0025300
     -,8X,'MOMENT',//)                                                  C0025400
      JW=0                                                              C0025500
      DO 505 X=0.0,SEGA+.005,SEGA/STEP                                  C0025600
      PROD(3)=SQRT(PLOAD/(ESMC*(WIDTH*THICK**3./12.)))                  C0025700
      PROD(1)=YO*(.5*(EXP(PROD(3)*X)-EXP(-PROD(3)*X)))                  C0025800
      PROD(2)=.5*(EXP(PROD(3)*SEGA)-EXP(-PROD(3)*SEGA))                 C0025900
      T(1)=PROD(1)/PROD(2)                                              C0026000
      T(2)=YO*PROD(3)/PROD(2)*(.5*(EXP(PROD(3)*X)+EXP                   C0026100
     -(-PROD(3)*X)))                                                    C0026200
C-----CALCULATION OF TOP AND BOTTOM FIBER STRESSES                      C0026220
      SIGT=-PLOAD*T(1)*THICK/2./(1./12.*WIDTH*THICK**3)+PLOAD/          C0026250
     -(WIDTH*THICK)                                                     C0026251
      SIGB=PLOAD*T(1)*THICK/2./(1./12.*WIDTH*THICK**3)+PLOAD/           C0026252
     -(WIDTH*THICK)                                                     C0026253
      J=JW+2                                                            C0026300
      IF(INT(J/2).NE.J/2.0) GO TO 199                                   C0026400
      WRITE(6,10) X, T(1), T(2), SIGT, PLOAD*T(1)                       C0026500
   10 FORMAT(10X,5(E11.4,3X))                                           C0026600
      WRITE(6,11) SIGB                                                  C0026650
   11 FORMAT(52X,E11.4,//)                                              C0026660
  199 JW=JW+1                                                           C0026700
  505 CONTINUE                                                          C0026800
C-                                                                      C0026900
C-----CALCULATION OF DEFLECTION AND SLOPE AS A FUNCTION                 C0027000
C-     OF ARC LENGTH FOR SEGMENT 2                                      C0027100
C-                                                                      C0027200
      WRITE(6,24)                                                       C0027300
   24 FORMAT(/////,' DEFLECTION AND SLOPE FOR SEGMENT 2',//)            C0027400
      WRITE(6,25)                                                       C0027500
   25 FORMAT(10X,'ARC LENGTH',5X,'DEFLECTION',7X,'SLOPE',10X,'STRESS'   C0027600
     -,12X,'MOMENT',11X,'SHEAR',//)                                     C0027700
      EXTERNAL FCN1                                                     C0027900
      R=WIDTH*THICK/(ALOG(RADO/RADI))                                   C0028000
      NW=2                                                              C0028100
      THETA=ARCOS((5.*THICK-BOND)/(6.*THICK))                           C0028200
      N=2                                                               C0028300
      YBAR=RADI+THICK/2.-R                                              C0028400
      X=0.0                                                             C0028500
      Y(1)=-T(1)                                                        C0028600
      Y(2)=-T(2)                                                        C0028700
      TOL=.000001                                                       C0028900
      IND=1                                                             C0029000
      J=0                                                               C0029100
      DO 510 Z=0.0,R*THETA+.001,R*THETA/STEP                            C0029200
      XEND=FLOAT(Z)                                                     C0029300
      CALL DVERK(N,FCN1,X,Y,XEND,TOL,IND,C,NW,W,IER)                    C0029400
      IF(IND.LT.0.OR.IER.GT.0) GO TO 511                                C0029500
      JW=J+10                                                           C0029600
      IF(INT(JW/10).NE.JW/10.0) GO TO 299                               C0029700
      WRITE(6,30) X,Y(1),Y(2), TORC, SHEAR                              C0029800
   30 FORMAT(10X,3(E11.4,3X),20X,E11.4,5X,E11.4,/)                      C0029900
      YGLOB=-.5*THICK                                                   C0029940
      DO 660 H=-(RADO-R),R-RADI,THICK/10.                               C0029950
C-----STRESSES FROM THE MOMENT DISTRIBUTION SUPERIMPOSED                C0029955
C-----ON TENSILE STRESSES AS A FUNCTION OF BEAM THICKNESS               C0029957
      AREA=WIDTH*THICK                                                  C0029959
      BSIGX=TORC*H/((R-H)*WIDTH*THICK*YBAR)+PLOAD*COS(X/R)/AREA         C0029960
```

```
      WRITE(6,600) YGLOB, BSIGX                                          00029961
  600 FORMAT(52X,F4.2,1X,E11.4)                                          00029962
      YGLOB=YGLOB+THICK/10.                                              00029966
  660 CONTINUE                                                           00029968
  299 J=J+1                                                              00030000
  510 CONTINUE                                                           00030100
  511 CONTINUE                                                           00030200
C-                                                                       00030300
C-----CALCULATION OF DEFLECTION AND SLOPE FOR SEGMENT 3                  00030400
C-     AS A FUNCTION O- ARC LENGTH                                       00030500
C-                                                                       00030600
      WRITE(6,27)                                                        00030700
   27 FORMAT(/////,' DEFLECTION AND SLOPE FOR SEGMENT 3',//)             00030800
      WRITE(6,26)                                                        00030900
   26 FORMAT(10X,'ARC LENGTH',5X,'DEFLECTION',7X,'SLOPE',10X,'STRESS'    00031000
     -,12X,'MOMENT',11X,'SHEAR',//)                                      00031100
      EXTERNAL FCN2                                                      00031200
      X=0.0                                                              00031300
      Y(1)=-Y(1)                                                         00031400
      Y(2)=-Y(2)                                                         00031500
      NW=2                                                               00031700
      N=2                                                                00031800
      IND=1                                                              00031900
      DO 515 M=1,24                                                      00032000
      C(M)=0.0                                                           00032100
  515 CONTINUE                                                           00032200
      J=0                                                                00032300
      DO 520 Z=0.0,R*THETA+.001 ,R*THETA/STEP                            00032400
      XEND=FLOAT(Z)                                                      00032500
      CALL DVERK(N,FCN2,X,Y,XEND,TOL,IND,C,NW,W,IER)                     00032600
      IF(IND.LT.0.OR.IER.GT.0) GO TO 525                                 00032700
      JW=J+10                                                            00032800
      IF(INT(JW/10).NE.JW/10.0) GO TO 349                                00032900
      WRITE(6,60) X, Y(1), Y(2), TORCS, SHEAR                            00033000
   60 FORMAT(10X,3(E11.4,3X),20X,E11.4,5X,E11.4,/)                       00033100
      YGLOB=-.5*THICK                                                    00033140
      DO 770 H=-(RADO-R),R-RADI,THICK/10.                                00033150
C-----STRESSES FROM THE MOMENT DISTRIBUTION SUPERIMPOSED                 00033155
C-----ON TENSILE STRESSES AS A FUNCTION OF BEAM THICKNESS                00033157
      CSIGX=-TORCS*H/((R-H)*WIDTH*THICK*YBAR)+PLOAD*COS(THETA-X/R)/      00033160
     -(WIDTH*THICK)                                                      00033161
      WRITE(6,700) YGLOB, CSIGX                                          00033162
  700 FORMAT(52X,F4.2,1X,E11.4)                                          00033164
      YGLOB=YGLOB+THICK/10.                                              00033166
  770 CONTINUE                                                           00033168
  349 J=J+1                                                              00033200
  520 CONTINUE                                                           00033300
  525 CONTINUE                                                           00033400
C-                                                                       00033600
C-----CALCULATION OF DEFLECTION AND SLOPE AS A FUNCTION                  00033700
C-     OF X FOR SEGMENT 4                                                00033800
C-                                                                       00033900
      WRITE(6,28)                                                        00034000
   28 FORMAT(/////,' DEFLECTION AND SLOPE FOR SEGMENT 4',//)             00034100
      WRITE(6,29)                                                        00034200
   29 FORMAT(10X,'X DISTANCE',5X,'DEFLECTION',7X,'SLOPE',//)             00034300
      EXTERNAL FCN3                                                      00034400
      X=0.0                                                              00034500
      NW=2                                                               00034600
      N=2                                                                00034700
      IND=1                                                              00034800
      DO 530 M=1,24                                                      00034900
```

```
      C(M)=0.0                                                          00035000
  530 CONTINUE                                                          00035100
      J=0                                                               00035200
      DO 535 Z=0.0,CONTA+.005,CONTA/STEP                                00035300
      XEND=FLOAT(Z)                                                     00035400
      CALL DVERK(N,FCN3,X,Y,XEND,TOL,IND,C,NW,W,IER)                    00035500
      IF(IND.LT.0.OR.IER.GT.0) GO TO 540                                00035600
      JW=J+10                                                           00035700
      IF(INT(JW/10).NE.JW/10.0) GO TO 399                               00035800
      WRITE(6,62) X, Y(1), Y(2)                                         00035900
   62 FORMAT(10X,3(E11.4,3X),/)                                         00036000
  399 J=J+1                                                             00036100
  535 CONTINUE                                                          00036200
  540 CONTINUE                                                          00036300
C-                                                                      00036400
C-----CALCULATION OF DEFLECTION AND SLOPE AS A FUNCTION OF              00036500
C-      X FOR SEGMENT 5                                                 00036600
C-                                                                      00036700
      WRITE(6,31)                                                       00036800
   31 FORMAT(/////,' DEFLECTION AND SLOPE FOR SEGMENT 5',//)            00036900
      WRITE(6,32)                                                       00037000
   32 FORMAT(10X,'X DISTANCE',5X,'DEFLECTION',7X,'SLOPE',10X,'STRESS'   00037100
     -,12X,'MOMENT',//)                                                 00037150
      EXTERNAL FCN4                                                     00037200
      X=0.0                                                             00037300
      NW=2                                                              00037400
      N=2                                                               00037500
      IND=1                                                             00037600
      DO 545 M=1,24                                                     00037700
      C(M)=0.0                                                          00037800
  545 CONTINUE                                                          00037900
      J=0                                                               00038000
      DO 550 Z=0.0,SEGB+.005,SEGB/STEP                                  00038100
      XEND=FLOAT(Z)                                                     00038200
      CALL DVERK(N,FCN4,X,Y,XEND,TOL,IND,C,NW,W,IER)                    00038300
      IF(IND.LT.0.OR.IER.GT.0) GO TO 555                                00038400
      JW=J+10                                                           00038500
      IF(X.EQ..15) GO TO 244                                            00038550
      IF(INT(JW/10).NE.JW/10.0) GO TO 245                               00038600
  244 WRITE(6,65) X, Y(1), Y(2), PLOAD*Y(1)                             00038700
   65 FORMAT(10X,3(E11.4,3X),20X,E11.4,/)                               00038800
      DO 880 H=-THICK/2.,THICK/2.,THICK/10.                             00038840
C-----STRESSES FROM THE MOMENT DISTRIBUTION SUPERIMPOSED                00038850
C-----ON TENSILE STRESSES AS A FUNCTION OF BEAM THICKNESS               00038855
      ESIGX=-PLOAD*Y(1)*H/(1./12.*WIDTH*THICK**3)+PLOAD/(WIDTH*THICK)   00038860
      WRITE(6,800) H, ESIGX                                             00038880
  800 FORMAT(52X,F4.2,1X,E11.4)                                         00038890
  880 CONTINUE                                                          00038895
  245 J=J+1                                                             00038900
  550 CONTINUE                                                          00039000
  555 CONTINUE                                                          00039100
      STOP                                                              00039300
      END                                                               00039400
C*************************************************************          00039410
C*************************************************************          00039420
C***                    SUBROUTINES                          ***          00039430
C*************************************************************          00039440
C*************************************************************          00039450
C-                                                                      00039460
C-----NUMERICAL INTEGRATION OF SEG2                                     00039470
C-                                                                      00039480
      SUBROUTINE FCN1(N,X,Y,YPRIME)                                     00039500
```

```
      COMMON R, PLOAD, ESMC, WIDTH, THICK, YBAR, THETA, EADH, PI          00039600
     -, BOND, J, TORC, TORCS, TRAC, TRACE, SHEAR                          00039700
      DIMENSION Y(2), YPRIME(2)                                           00039800
      YPRIME(1)=Y(2)                                                      00039900
C-                                                                        00039950
C-----ECCENTRICITY DUE ONLY TO EXTENSIONAL EFFECTS                        00039955
C-                                                                        00039960
      YECC=PLOAD*X*COS(X/R)*SIN(X/R)/(WIDTH*THICK*ESMC)                   00040000
      TORC=PLOAD*(YECC*TRAC+YBAR+R*(1.-COS(X/R))-Y(1)*COS(X/R))           00040100
      SHEAR=PLOAD*(PLOAD/(WIDTH*THICK*ESMC)*(X/R*COS(X/R)**2              00040150
     -+SIN(X/R)*(X/R*(-SIN(X/R))+COS(X/R)))+SIN(X/R)+Y(1)/R               00040160
     -*SIN(X/R))                                                          00040170
  560 YPRIME(2)=(R+Y(1))*TORC/(R**2.*ESMC*WIDTH*THICK*YBAR)               00040500
      RETURN                                                              00040600
      END                                                                 00040700
C-                                                                        00040750
C-----NUMERICAL INTEGRATION OF SEG3                                       00040755
C-                                                                        00040760
      SUBROUTINE FCN2(N,X,Y,YPRIME)                                       00040800
      COMMON R, PLOAD, ESMC, WIDTH, THICK, YBAR, THETA, EADH, PI          00040900
     -, BOND, J, TORC, TORCS, TRAC, TRACE, SHEAR                          00041000
      DIMENSION Y(2), YPRIME(2)                                           00041100
      YPRIME(1)=Y(2)                                                      00041200
C-                                                                        00041250
C-----ECCENTRICITY DUE TO GEOMETRY                                        00041260
C-                                                                        00041270
      AEGEO=YBAR+R*(1.-COS(THETA))-2.*YBAR*COS(THETA)                     00041300
C-                                                                        00041350
C-----ECCENTRICITY DUE TO GEOMETRY                                        00041360
C-                                                                        00041370
      BEGEO=R*(SIN(X/R+PI/2.-THETA)-SIN(PI/2.-THETA))                     00041400
C-                                                                        00041450
C-----ECCENTRICITY DUE ONLY TO EXTENSIONAL EFFECTS                        00041460
C-                                                                        00041470
      EEXT=PLOAD*(SIN(PI/2.-THETA+X/R))*SIN(THETA-X/R)*X/(                00041500
     -WIDTH*THICK*ESMC)                                                   00041600
C-                                                                        00041650
C-----ECCENTRICITY DUE TO DEFLECTION                                      00041660
C-                                                                        00041670
      EDEFL=Y(1)*COS(THETA-X/R)                                           00041700
      TORCS=PLOAD*(AEGEO+BEGEO+EDEFL+EEXT*TRACE)                          00041800
      SHEAR=PLOAD*(COS(X/R+PI/2.-THETA)-Y(1)/R*SIN(PI/2.                  00041850
     --THETA+X/R)+PLOAD*((SIN(PI/2.-THETA+X/R))*(SIN(THETA-               00041860
     -X/R)/(WIDTH*THICK*ESMC)-X/(WIDTH*THICK*R*ESMC)*COS                  00041870
     -(THETA-X/R))+X/(WIDTH*THICK*ESMC*R)*SIN(THETA-X/R)                  00041880
     -*COS(PI/2.-THETA+X/R)))                                             00041890
      YPRIME(2)=(R+Y(1))*TORCS/(R**2.*ESMC*WIDTH*THICK*YBAR)              00042000
      RETURN                                                              00042100
      END                                                                 00042200
C-                                                                        00042250
C-----NUMERICAL INTEGRATION OF SEG4                                       00042260
C-                                                                        00042270
      SUBROUTINE FCN3(N,X,Y,YPRIME)                                       00042300
      COMMON R, PLOAD, ESMC, WIDTH, THICK, YBAR, THETA, EADH, PI          00042400
     -, BOND                                                              00042500
      DIMENSION Y(2), YPRIME(2), EI(3)                                    00042600
      EI(1)=ESMC*(1./12.*THICK**3.*WIDTH+WIDTH*THICK*(.5*THICK+BOND       00042700
     -/2.)**2.)                                                           00042800
      EI(2)=EADH*1./12.*THICK**3*WIDTH                                    00042900
      EI(3)=ESMC*(1./12.*WIDTH*THICK**3+WIDTH*THICK*(.5*THICK+BOND/       00043000
     -2.)**2.)                                                            00043100
      YPRIME(1)=Y(2)                                                      00043200
```

```
      DEN=EI(1)+EI(2)+EI(3)                                              00043300
      YPRIME(2)=PLOAD*(.5*THICK+BOND/2.+Y(1))/DEN                        00043400
      RETURN                                                             00043500
      END                                                                00043600
C-                                                                       00043650
C-----NUMERICAL INTEGRATION OF SEG5                                      00043660
C-                                                                       00043670
      SUBROUTINE FCN4(N,X,Y,YPRIME)                                      00043700
      COMMON R, PLOAD, ESMC, WIDTH, THICK, YBAR, THETA, EADH, PI         00043800
     -, BOND                                                             00043900
      DIMENSION Y(2), YPRIME(2)                                          00044000
      YPRIME(1)=Y(2)                                                     00044100
      YPRIME(2)=PLOAD*(Y(1))/(ESMC*WIDTH*THICK**3./12.)                  00044200
      RETURN                                                             00044300
      END                                                                00044400
```

b. CONVERT

The program CONVERT essentially performs the tedious calculations involved in computing the boundary conditions for the finite-element model. Stresses dictated by the beam bending model are converted to equivalent point forces which are then applied to the finely meshed ends of the finite-element structure. In converting the stress distribution from deformed to undeformed geometry the program insures that the model be maintained in equilibrium through the introduction of a correcting moment.

The important parameters utilized in the routine are defined in the nomenclature section of the program. Frequent comment cards are intended to assist the user in the utilization of the program.

```
C-     CCCCC   OOOOO   N   N   V   V   EEEEE   RRRRR   TTTTT                00000004
C-     C       O   O   NN  N   V   V   E       R   R     T                  00000005
C-     C       O   O   N N N   V   V   EEE     RRRRR     T                  00000006
C-     C       O   O   N  NN    V V    E       R R       T                  00000007
C-     CCCCC   OOOOO   N   N     V     EEEEE   R  R      T                  00000008
C-                                                                          00000010
C-                                                                          00000011
C-                                                                          00000012
C-                                                                          00000013
C-                                                                          00000014
C-                                                                          00000015
C-                  STRESS TRANSFORMATION PROGRAM                           00000016
C-                                                                          00000017
C-                     DEVELOPED BY:  RICHARD C. GIVLER                     00000018
C-                         UNIVERSITY OF DELAWARE                           00000019
C-                            OCT 78 - JAN 79                               00000020
C-                                                                          00000030
C-                                                                          00000031
C-                                                                          00000032
C-                                                                          00000033
C-                                                                          00000034
C-                                                                          00000035
C-                                                                          00000036
C-                                                                          00000037
$RESET FREE                                                                 00000100
      DIMENSION ZOM(5), RES(5), PART(4), ZOMX(5), RESB(5)                   00000200
     -, RESX(5), RESY(5), ZOMA(5), ZOMB(5), BMOM(5), ZOMXB(5)               00000300
     -, RESXB(5), RESYB(5)                                                  00000400
C-                                                                          00000420
C-----PARAMETERS AND NOMENCLATURE                                           00000430
C-                                                                          00000440
C------MATERIAL THICKNESS                                                  00000470
      THICK=.1                                                              00000500
C------MATERIAL WIDTH                                                      00000550
      WIDTH=1.                                                              00000600
C------ROTATION OF LEFT HAND FACE FROM UNDEFORMED GEOMETRY (RAD)           00000650
      DUDSA=-.01642                                                         00000700
C------ROTATION OF RIGHT HAND FACE FROM UNDEFORMED GEOMETRY (RAD)          00000750
      DUDSB=.008420                                                         00000800
C------TOTAL MOMENT ON LEFT HAND FACE (IN. LBS.)                           00000850
      TOTMOA=10.64                                                          00000900
C------SHEAR ON LEFT HAND FACE                                             00000925
      SHEARA=132.6                                                          00000950
C------TOTAL MOMENT ON RIGHT HAND FACE (IN. LBS.)                          00000975
      TOTMOB=-1.968                                                         00001000
      AREA=WIDTH*THICK                                                      00001100
C------OUTSIDE RADIUS IN INCHES                                            00001150
      RADO=3.5*THICK                                                        00001200
C------INSIDE RADIUS IN INCHES                                             00001250
      RADI=2.5*THICK                                                        00001300
C------RADIUS OF CURVATURE OF CURVED MEMBERS                               00001350
      R=WIDTH*THICK/(ALOG(RADO/RADI))                                       00001400
C------DIFFERENCE BETWEEN NEUTRAL AXIS AND CENTROIDAL AXIS (IN.)           00001450
      YBAR=RADI+THICK/2.-R                                                  00001500
C------ADHESIVE BOND THICKNESS IN INCHES                                   00001550
      BOND=.03                                                              00001600
C------ANGLE SUBTENDED BY CURVED MEMBERS IN RADIANS                        00001650
      THETA=ARCOS((5.*THICK-BOND)/(6.*THICK))                               00001700
C------TENSILE LOAD                                                        00001750
      PLOAD=200.                                                            00001800
C-                                                                          00001850
```

```
C------NOTE:  DEFLECTIONS ARE MEASURED NORMAL TO THE NEUTRAL AXIS        00001852
C-                                                                      00001854
C------DEFLECTION OF LEFT HAND FACE (IN.)                               00001870
      DEFLA=-.01547                                                     00001900
C------DEFLECTION OF RIGHT HAND FACE (IN.)                              00001970
      DEFLB=-.007872                                                    00002000
C------                                                                 00002010
C-*********************************************************            00002020
C------                                                                 00002100
C------RESOLVING STRESS DISTRIBUTION ON LEFT HAND FACE                   00002200
C------                                                                 00002300
C-*********************************************************            00002320
C------                                                                 00002400
C------                                                                 00002500
C------CALCULATION OF RESULTANT POINT FORCES FROM THE                    00002600
C------STRESS DISTRIBUTION                                              00002700
C------                                                                 00002800
      H=R-RADI                                                          00002900
      DO 100 N=1,5                                                      00003000
      PART(1)=TOTMOA/(YBAR*AREA)*(R-H-R*ALOG(R-H))                      00003100
     -+PLOAD/AREA*COS(THETA+DUDSA)*H                                    00003200
      H=H-.02                                                           00003300
      PART(2)=TOTMOA/(YBAR*AREA)*(R-H-R*ALOG(R-H))                      00003400
     -+PLOAD/AREA*COS(THETA+DUDSA)*H                                    00003500
      RES(N)=PART(1)-PART(2)                                            00003600
  100 CONTINUE                                                          00003800
C------                                                                 00003900
C------CALCULATION OF ACTUAL MOMENTS FROM THE STRESS DISTRIBUTION       00004000
C------                                                                 00004100
      H=R-RADI                                                          00004200
      DO 200 N=1,5                                                      00004300
      PART(3)=TOTMOA/(YBAR*AREA)*(-1.)*(.5*(R-H)**2-2.*R*(R-H)+R**2     00004400
     -*ALOG(R-H))+.5*PLOAD/AREA*COS(THETA+DUDSA)*H**2                   00004600
      H=H-.02                                                           00004700
      PART(4)=TOTMOA/(YBAR*AREA)*(-1.)*(.5*(R-H)**2-2.*R*(R-H)+R**2     00004800
     -*ALOG(R-H))+.5*PLOAD/AREA*COS(THETA+DUDSA)*H**2                   00004900
      ZOM(N)=PART(3)-PART(4)                                            00005000
  200 CONTINUE                                                          00005200
C------                                                                 00005300
C------CALCULATION OF CORRECTION MOMENT DUE TO THE                      00005400
C------REPRESENTATION OF THE STRESS DISTRIBUTION BY POINT               00005500
C------FORCES                                                          00005600
C------                                                                 00005700
      H=R-RADI-.01                                                      00005800
      DO 300 N=1,5                                                      00005900
      ZOMX(N)=ZOM(N)-RES(N)*H                                           00006000
      H=H-.02                                                           00006200
  300 CONTINUE                                                          00006300
C------                                                                 00006400
C------CALCULATION OF MOMENT DUE TO TRANSLATION OF THE STRESS           00006500
C------DISTRIBUTION THROUGH SPACE FROM THE DEFORMED GEOMETRY            00006600
C------TO THE UNDEFORMED GEOMETRY                                       00006700
C------                                                                 00006800
C------                                                                 00006900
C------CALCULATION OF MOMX FOR INPUT INTO THE FINITE ELEMENT            00007000
C------MODEL                                                           00007100
C------                                                                 00007200
      H=R-RADI-.01                                                      00007300
      DO 400 N=1,5                                                      00007400
      ZOMA(N)=-RES(N)*((H+DEFLA)*COS(DUDSA)-H)                          00007500
      ZOMX(N)=ZOMX(N)+ZOMA(N)                                           00007700
      H=H-.02                                                           00007900
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  400 CONTINUE                                                          00008000
C------                                                                 00008100
C------CALCULATION OF FY AND FZ FOR INPUT INTO THE FINITE               00008200
C------ELEMENT MODEL                                                   00008300
C------                                                                 00008400
      WRITE(6,25)                                                       00008500
   25 FORMAT(///////,6X,'BOUNDARY CONDITIONS FOR  LEFT HAND SEGMENT',/) 00008600
      WRITE(6,30)                                                       00008700
   30 FORMAT(/,4X,'NODE',6X,'FY',13X,'FZ',12X,'MOMX',//)                00008800
      DO 500 N=1,5                                                      00008900
      RESX(N)=-RES(N)*COS(THETA+DUDSA)-SHEARA/5.*SIN(THETA)             00009000
      RESY(N)=-RES(N)*SIN(THETA+DUDSA)+SHEARA/5.*COS(THETA)             00009100
      WRITE(6,2) N, RESX(N), RESY(N), -ZOMX(N)                          00009200
    2 FORMAT(5X,F2.0,3X,3(E10.4,5X),/)                                  00009300
  500 CONTINUE                                                          00009400
C------                                                                 00009500
C------                                                                 00009600
C*********************************************************             00009700
C------                                                                 00009800
C------RESOLVING STRESS DISTRIBUTION ON RIGHT HAND FACE                 00009900
C------                                                                 00010000
C*********************************************************             00010100
C------                                                                 00010200
C------                                                                 00010300
C------                                                                 00010400
C------CALCULATION OF RESULTANT POINT FORCES FROM THE STRESS            00010500
C------DISTRIBUTION                                                    00010600
C------                                                                 00010700
      H=THICK/2.                                                        00010800
      DO 1000 N=1,5                                                     00010900
      PART(1)=-.5*TOTMOB*H**2/(1./12.*WIDTH*THICK**3)+PLOAD/AREA*H      00011000
      H=H-.02                                                           00011100
      PART(2)=-.5*TOTMOB*H**2/(1./12.*WIDTH*THICK**3)+PLOAD/AREA*H      00011200
      RESB(N)=PART(1)-PART(2)                                           00011300
 1000 CONTINUE                                                          00011500
C------                                                                 00011600
C------CALCULATION OF ACTUAL MOMENTS FROM THE STRESS                   00011700
C------DISTRIBUTION                                                    00011800
C------                                                                 00011900
      H=THICK/2.                                                        00012000
      DO 1100 N=1,5                                                     00012100
      PART(3)=1./3.*(TOTMOB)*H**3/(1./12.*WIDTH*THICK**3)               00012200
     --.5*PLOAD/AREA*H**2                                               00012300
      H=H-.02                                                           00012350
      PART(4)=1./3.*(TOTMOB)*H**3/(1./12.*WIDTH*THICK**3)               00012400
     --.5*PLOAD/AREA*H**2                                               00012500
      BMOM(N)=PART(3)-PART(4)                                           00012600
 1100 CONTINUE                                                          00012800
C------                                                                 00012900
C------CALCULATION OF CORRECTION MOMENT DUE TO THE                     00013000
C------REPRESENTATION OF THE STRESS DISTRIBUTION BY POINT              00013100
C------FORCES                                                         00013200
C------                                                                 00013300
      H=THICK/2.-.01                                                    00013400
      DO 1200 N=1,5                                                     00013500
      ZOMXB(N)=BMOM(N)+RESB(N)*H                                        00013600
      H=H-.02                                                           00013750
 1200 CONTINUE                                                          00013800
C------                                                                 00013900
C------CALCULATION OF THE MOMENT DUE TO TRANSLATING THE STRESS          00014000
C------DISTRIBUTION FROM DEFORMED TO UNDEFORMED GEOMETRY                00014100
C------                                                                 00014200
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C------                                                          00014300
C------CALCULATION OF MOMENT MX FOR INPUT INTO THE FINITE          00014400
C------ELEMENT MODEL                                            00014500
C------                                                          00014600
      H=THICK/2.-.01                                             00014700
      DO 1300 N=1,5                                              00014800
      ZOMB(N)=RESB(N)*((H-DEFLB)*COS(DUDSB)-H)                   00014900
      ZOMXB(N)=ZOMXB(N)+ZOMB(N)                                  00015100
      H=H-.02                                                    00015200
 1300 CONTINUE                                                   00015300
C------                                                          00015400
C------CALCULATION OF FY AND FZ FOR INPUT INTO THE FINITE        00015500
C------ELEMENT MODEL                                            00015600
C------                                                          00015700
      WRITE(6,40)                                                00015800
   40 FORMAT(//////,10X,'BONDARY CONDITIONS FOR SEGB',/)         00015900
      WRITE(6,45)                                                00016000
   45 FORMAT(/,4X,'NODE',6X,'FY',13X,'FZ',12X,'MOMX',//)         00016100
      DO 1400 N=1,5                                              00016200
      RESXB(N)=RESB(N)*COS(DUDSB)                                00016300
      RESYB(N)=RESB(N)*SIN(DUDSB)                                00016400
      WRITE(6,20) N, RESXB(N), RESYB(N), ZOMXB(N)                00016500
   20 FORMAT(5X,F2.0,3X,3(E10.4,5X),/)                           00016600
 1400 CONTINUE                                                   00016700
      END                                                        00016800
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      SUBROUTINE DVERK  (N,FCN,X,Y,XEND,TOL,IND,C,NW,W,IER)                 DVEK0010
C                                                                           DVEK0020
C-DVERK--------S-------LIBRARY 3-----------------------------------------DVEK0030
C                                                                           DVEK0040
C   FUNCTION            - SOLUTION OF A SYSTEM OF FIRST ORDER ORDINARY      DVEK0050
C                           DIFFERENTIAL EQUATIONS OF THE FORM              DVEK0060
C                           DY/DX = F(X,Y) WITH INITIAL CONDITIONS.         DVEK0070
C                           A RUNGE-KUTTA METHOD BASED ON VERNERS FIFTH     DVEK0080
C                           AND SIXTH ORDER PAIR OF FORMULAS IS USED.       DVEK0090
C   USAGE               - CALL DVERK(N,FCN,X,Y,XEND,TOL,IND,C,NW,W,IER)     DVEK0100
C   PARAMETERS    N     - NUMBER OF EQUATIONS. (INPUT)                      DVEK0110
C                 FCN   - NAME OF SUBROUTINE FOR EVALUATING FUNCTIONS.      DVEK0120
C                           (INPUT)                                         DVEK0130
C                           THE SUBROUTINE ITSELF MUST ALSO BE PROVIDED     DVEK0140
C                             BY THE USER AND IT SHOULD BE OF THE           DVEK0150
C                             FOLLOWING FORM                                DVEK0160
C                               SUBROUTINE FCN(N,X,Y,YPRIME)                DVEK0170
C                               DIMENSION Y(N),YPRIME(N)                    DVEK0180
C                                     .                                     DVEK0190
C                                     .                                     DVEK0200
C                                     .                                     DVEK0210
C                           FCN SHOULD EVALUATE YPRIME(1),....,YPRIME(N)    DVEK0220
C                             GIVEN N,X, AND Y(1),....,Y(N).  YPRIME(I)     DVEK0230
C                             IS THE FIRST DERIVATIVE OF Y(I) WITH          DVEK0240
C                             RESPECT TO X.                                 DVEK0250
C                           FCN MUST APPEAR IN AN EXTERNAL STATEMENT IN     DVEK0260
C                             THE CALLING PROGRAM AND N,X,Y(1),....,Y(N)    DVEK0270
C                             MUST NOT BE ALTERED BY FCN.                   DVEK0280
C                 X     - INDEPENDENT VARIABLE. (INPUT AND OUTPUT)          DVEK0290
C                           ON INPUT, X SUPPLIES THE INITIAL VALUE.         DVEK0300
C                           ON OUTPUT, X IS REPLACED WITH XEND UNLESS       DVEK0310
C                             ERROR CONDITIONS ARISE.  SEE THE DES-         DVEK0320
C                             CRIPTION OF PARAMETER IND.                    DVEK0330
C                 Y     - DEPENDENT VARIABLES, VECTOR OF LENGTH N.          DVEK0340
C                           (INPUT AND OUTPUT)                              DVEK0350
C                           ON INPUT, Y(1),....,Y(N) SUPPLY INITIAL         DVEK0360
C                             VALUES.                                       DVEK0370
C                           ON OUTPUT, Y(1),....,Y(N) ARE REPLACED WITH     DVEK0380
C                             AN APPROXIMATE SOLUTION AT XEND UNLESS        DVEK0390
C                             ERROR CONDITIONS ARISE.  SEE THE DES-         DVEK0400
C                             CRIPTION OF PARAMETER IND.                    DVEK0410
C                 XEND  - VALUE OF X AT WHICH SOLUTION IS DESIRED.          DVEK0420
C                           (INPUT)                                         DVEK0430
C                           XEND MAY BE LESS THAN THE INITIAL VALUE OF      DVEK0440
C                             X.                                            DVEK0450
C                 TOL   - TOLERANCE FOR ERROR CONTROL. (INPUT)              DVEK0460
C                           THE SUBROUTINE ATTEMPTS TO CONTROL A NORM       DVEK0470
C                             OF THE LOCAL ERROR IN SUCH A WAY THAT THE     DVEK0480
C                             GLOBAL ERROR IS PROPORTIONAL TO TOL.          DVEK0490
C                             MAKING TOL SMALLER IMPROVES ACCURACY AND      DVEK0500
C                             MORE THAN ONE RUN, WITH DIFFERENT VALUES      DVEK0510
C                             OF TOL, CAN BE USED IN AN ATTEMPT TO          DVEK0520
C                             ESTIMATE THE GLOBAL ERROR.                    DVEK0530
C                           IN THE DEFAULT CASE (IND=1), THE GLOBAL         DVEK0540
C                             ERROR IS                                      DVEK0550
C                                 MAX(ABS(E(1)),....,ABS(E(N)))             DVEK0560
C                             WHERE E(K)=(Y(K)-YT(K))/MAX(1,ABS(Y(K)))      DVEK0570
C                             YT(K) IS THE TRUE SOLUTION, AND               DVEK0580
C                             Y(K) IS THE COMPUTED SOLUTION AT XEND.        DVEK0590
C                             FOR K=1,2,....,N.                             DVEK0600
C                             OTHER ERROR CONTROL OPTIONS ARE AVAILABLE.    DVEK0610
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C                SEE THE DESCRIPTION OF PARAMETERS IND AND      DVEK0620
C                C BELOW.                                       DVEK0630
C          IND   - INDICATOR. (INPUT AND OUTPUT)                DVEK0640
C                  ON INITIAL ENTRY IND MUST BE SET EQUAL TO    DVEK0650
C                  EITHER 1 OR 2.                               DVEK0660
C                  IND = 1 CAUSES ALL DEFAULT OPTIONS TO BE     DVEK0670
C                    USED AND ELIMINATES THE NEED TO SET        DVEK0680
C                    SPECIFIC VALUES IN THE COMMUNICATIONS      DVEK0690
C                    VECTOR C.                                  DVEK0700
C                  IND = 2 ALLOWS OPTIONS TO BE SELECTED.  IN   DVEK0710
C                    THIS CASE, THE FIRST 9 COMPONENTS OF C     DVEK0720
C                    MUST BE INITIALIZED TO SELECT OPTIONS AS   DVEK0730
C                    DESCRIBED BELOW.                           DVEK0740
C                THE SUBROUTINE WILL NORMALLY RETURN WITH       DVEK0750
C                  IND = 3, HAVING REPLACED THE INITIAL VALUES  DVEK0760
C                  OF X AND Y WITH, RESPECTIVELY, THE VALUE     DVEK0770
C                  XEND AND AN APPROXIMATION TO Y AT XEND.      DVEK0780
C                THE SUBROUTINE CAN BE CALLED REPEATEDLY WITH   DVEK0790
C                  NEW VALUES OF XEND WITHOUT CHANGING ANY      DVEK0800
C                  OF THE OTHER PARAMETERS.                     DVEK0810
C                THREE ERROR RETURNS ARE ALSO POSSIBLE.  IN     DVEK0820
C                  WHICH CASE X AND Y WILL BE THE MOST          DVEK0830
C                  RECENTLY ACCEPTED VALUES.                    DVEK0840
C                  IND = -3 INDICATES THAT THE SUBROUTINE WAS   DVEK0850
C                    UNABLE TO SATISFY THE ERROR REQUIREMENT.   DVEK0860
C                    THIS MAY MEAN THAT TOL IS TOO SMALL.       DVEK0870
C                  IND = -2 INDICATES THAT THE VALUE OF HMIN    DVEK0880
C                    (MINIMUM STEP-SIZE) IS GREATER THAN HMAX   DVEK0890
C                    (MAXIMUM STEP-SIZE), WHICH PROBABLY MEANS  DVEK0900
C                    THAT THE REQUESTED TOL (WHICH IS USED IN   DVEK0910
C                    THE CALCULATION OF HMIN) IS TOO SMALL.     DVEK0920
C                  IND = -1 INDICATES THAT THE ALLOWED MAXIMUM  DVEK0930
C                    NUMBER OF FCN EVALUATIONS HAS BEEN         DVEK0940
C                    EXCEEDED.  THIS CAN ONLY OCCUR IF OPTION   DVEK0950
C                    C(7), AS DESCRIBED BELOW, HAS BEEN USED.   DVEK0960
C          C     - COMMUNICATIONS VECTOR OF LENGTH 24. (INPUT IF DVEK0970
C                  IND.NE.1, AND OUTPUT).                       DVEK0980
C                  C IS USED TO SELECT OPTIONS AND TO RETAIN    DVEK0990
C                    INFORMATION BETWEEN CALLS.  THE USER NEED  DVEK1000
C                    NOT BE CONCERNED WITH THE FOLLOWING        DVEK1010
C                    DESCRIPTION OF THE ELEMENTS OF C WHEN      DVEK1020
C                    DEFAULT OPTIONS ARE USED (IND=1).          DVEK1030
C                    HOWEVER, WHEN IT IS DESIRED TO USE IND=2   DVEK1040
C                    AND SELECT OPTIONS, A BASIC UNDERSTANDING  DVEK1050
C                    OF DVERK IS REQUIRED.  THE FOLLOWING       DVEK1060
C                    PARAGRAPH DESCRIBES, BRIEFLY, THE BASIC    DVEK1070
C                    TERMS.  FOR MORE DETAILS, SEE THE          DVEK1080
C                    REFERENCE.                                 DVEK1090
C                    DVERK ADVANCES THE INDEPENDENT VARIABLE    DVEK1100
C                    X ONE STEP AT A TIME UNTIL XEND IS         DVEK1110
C                    REACHED.  THE SOLUTION IS COMPUTED AT      DVEK1120
C                    XTRIAL = X+HTRIAL ALONG WITH AN ERROR      DVEK1130
C                    ESTIMATE EST.  IF EST IS LESS THAN OR      DVEK1140
C                    EQUAL TO TOL (SUCCESSFUL STEP), THE STEP   DVEK1150
C                    IS ACCEPTED AND X IS ADVANCED TO XTRIAL.   DVEK1160
C                    IF EST IS GREATER THAN TOL (FAILURE)       DVEK1170
C                    HTRIAL IS ADJUSTED AND THE SOLUTION IS     DVEK1180
C                    RECOMPUTED.  HMAG = ABS(HTRIAL) IS NEVER   DVEK1190
C                    ALLOWED TO EXCEED HMAX NOR IS IT ALLOWED   DVEK1200
C                    TO BECOME SMALLER THAN HMIN.  THE FIRST    DVEK1210
C                    TRIAL STEP IS HSTART.  DURING THE          DVEK1220
C                    COMPUTATION, A COUNTER (C(23)) IS          DVEK1230
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C                 INCREMENTED EACH TIME A TRIAL STEP FAILS  DVEK1240
C                 TO PROVIDE A SOLUTION SATISFYING THE ERRORDVEK1250
C                 TOLERANCE.  ANOTHER COUNTER (C(22)) IS    DVEK1260
C                 USED TO RECORD THE NUMBER OF SUCCESSFUL   DVEK1270
C                 STEPS.  AFTER A SUCCESSFUL STEP, C(23) IS DVEK1280
C                 SET TO ZERO.                              DVEK1290
C               OPTIONS.  IF THE SUBROUTINE IS ENTERED WITH DVEK1300
C                 IND=2, THE FIRST 9 COMPONENTS OF THE      DVEK1310
C                 COMMUNICATIONS VECTOR MUST BE INITIALIZED DVEK1320
C                 BY THE USER.  NORMALLY THIS IS DONE BY    DVEK1330
C                 FIRST SETTING THEM ALL TO ZERO, AND THEN  DVEK1340
C                 THOSE CORRESPONDING TO PARTICULAR OPTIONS DVEK1350
C                 ARE MADE NON-ZERO.                        DVEK1360
C     C(1)    - ERROR CONTROL INDICATOR.                    DVEK1370
C                 THE SUBROUTINE ATTEMPTS TO CONTROL A NORM DVEK1380
C                 OF THE LOCAL ERROR IN SUCH A WAY THAT THE DVEK1390
C                 GLOBAL ERROR IS PROPORTIONAL TO TOL.      DVEK1400
C                 THE DEFINITION OF GLOBAL ERROR FOR THE    DVEK1410
C                 DEFAULT CASE (IND=1) IS GIVEN IN THE      DVEK1420
C                 DESCRIPTION OF PARAMETER TOL.  THE DEFAULT DVEK1430
C                 WEIGHTS ARE 1/MAX(1,ABS(Y(K))).  WHEN IND=2 DVEK1440
C                 IS USED, THE WEIGHTS ARE DETERMINED       DVEK1450
C                 ACCORDING TO THE VALUE OF C(1).           DVEK1460
C                 IF C(1)=1 THE WEIGHTS ARE 1               DVEK1470
C                           (ABSOLUTE ERROR CONTROL)        DVEK1480
C                 IF C(1)=2 THE WEIGHTS ARE 1/ABS(Y(K))     DVEK1490
C                           FOR K=1,2,...,N.                DVEK1500
C                           (RELATIVE ERROR CONTROL)        DVEK1510
C                 IF C(1)=3 THE WEIGHTS ARE                 DVEK1520
C                           1/MAX(ABS(C(2)),ABS(Y(K)))      DVEK1530
C                           FOR K=1,2,...,N.                DVEK1540
C                           (RELATIVE ERROR CONTROL, UNLESS DVEK1550
C                           ABS(Y(K)) IS LESS THAN THE FLOOR DVEK1560
C                           VALUE,ABS(C(2)))                DVEK1570
C                 IF C(1)=4 THE WEIGHTS ARE                 DVEK1580
C                           1/MAX(ABS(C(K+30)),ABS(Y(K)))   DVEK1590
C                           FOR K=1,2,...,N.                DVEK1600
C                           (HERE INDIVIDUAL FLOOR VALUES   DVEK1610
C                           ARE USED)                       DVEK1620
C                           IN THIS CASE, THE DIMENSION OF C DVEK1630
C                           MUST BE GREATER THAN OR EQUAL TO DVEK1640
C                           N+30 AND C(31), C(32),...,C(N+30) DVEK1650
C                           MUST BE INITIALIZED BY THE USER. DVEK1660
C                 IF C(1)=5 THE WEIGHTS ARE 1/ABS(C(K+30))  DVEK1670
C                           FOR K=1,2,...,N.                DVEK1680
C                           IN THIS CASE, THE DIMENSION OF C DVEK1690
C                           MUST BE GREATER THAN OR EQUAL TO DVEK1700
C                           N+30 AND C(31), C(32),...,C(N+30) DVEK1710
C                           MUST BE INITIALIZED BY THE USER. DVEK1720
C                 FOR ALL OTHER VALUES OF C(1), INCLUDING   DVEK1730
C                    C(1)=0 THE DEFAULT VALUES OF           DVEK1740
C                           THE WEIGHTS ARE TAKEN TO BE     DVEK1750
C                           1/MAX(1,ABS(Y(K)))              DVEK1760
C                           FOR K=1,2,...,N.                DVEK1770
C     C(2)    - FLOOR VALUE.  USED WHEN THE INDICATOR C(1)  DVEK1780
C                 HAS THE VALUE 3.                          DVEK1790
C     C(3)    - HMIN SPECIFICATION.  IF NOT ZERO, THE SUB-  DVEK1800
C                 ROUTINE CHOOSES HMIN TO BE ABS(C(3)).     DVEK1810
C                 OTHERWISE IT USES THE DEFAULT VALUE       DVEK1820
C                 10*MAX(DWARF,RREB*MAX(NORM(Y)/TOL,ABS(X))) DVEK1830
C                 WHERE DWARF IS A VERY SMALL POSITIVE MACHINEDVEK1840
C                 NUMBER AND RREB IS THE RELATIVE ROUNDOFF  DVEK1850
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C                ERROR BOUND.                                          DVEK1860
C          C(4)  - HSTART SPECIFICATION.  IF NOT ZERO, THE SUB-        DVEK1870
C                  ROUTINE WILL USE AN INITIAL HMAG EQUAL TO           DVEK1880
C                  ABS(C(4)), EXCEPT OF COURSE FOR THE RE-             DVEK1890
C                  STRICTIONS IMPOSED BY HMIN AND HMAX.                DVEK1900
C                  OTHERWISE IT USES THE DEFAULT VALUE                 DVEK1910
C                    HMAX*(TOL)**(1/6).                                DVEK1920
C          C(5)  - SCALE SPECIFICATION.  THIS IS INTENDED TO BE        DVEK1930
C                  A MEASURE OF THE SCALE OF THE PROBLEM.              DVEK1940
C                  LARGER VALUES OF SCALE TEND TO MAKE THE             DVEK1950
C                  METHOD MORE RELIABLE, FIRST BY POSSIBLY RE-         DVEK1960
C                  STRICTING HMAX (AS DESCRIBED BELOW) AND             DVEK1970
C                  SECOND, BY TIGHTENING THE ACCEPTANCE                DVEK1980
C                  REQUIREMENT.  IF C(5) IS ZERO, A DEFAULT            DVEK1990
C                  VALUE OF 1 IS USED.  FOR LINEAR HOMOGENEOUS         DVEK2000
C                  PROBLEMS WITH CONSTANT COEFFICIENTS, AN             DVEK2010
C                  APPROPRIATE VALUE FOR SCALE IS A NORM OF            DVEK2020
C                  THE ASSOCIATED MATRIX.  FOR OTHER PROBLEMS,         DVEK2030
C                  AN APPROXIMATION TO AN AVERAGE VALUE OF A           DVEK2040
C                  NORM OF THE JACOBIAN ALONG THE TRAJEC-              DVEK2050
C                  TORY MAY BE APPROPRIATE.                            DVEK2060
C          C(6)  - HMAX SPECIFICATION.  FOUR CASES ARE POSSIBLE.       DVEK2070
C                  IF C(6).NE.0 AND C(5).NE.0, HMAX IS TAKEN           DVEK2080
C                    TO BE MIN(ABS(C(6)),2/ABS(C(5))).                 DVEK2090
C                  IF C(6).NE.0 AND C(5).EQ.0, HMAX IS TAKEN           DVEK2100
C                    TO BE ABS(C(6)).                                  DVEK2110
C                  IF C(6).EQ.0 AND C(5).NE.0, HMAX IS TAKEN           DVEK2120
C                    TO BE 2/ABS(C(5)).                                DVEK2130
C                  IF C(6).EQ.0 AND C(5).EQ.0, HMAX IS GIVEN           DVEK2140
C                    A DEFAULT VALUE OF 2.                             DVEK2150
C          C(7)  - MAXIMUM NUMBER OF FUNCTION EVALUATIONS.  IF         DVEK2160
C                  NOT ZERO, AN ERROR RETURN WITH IND = -1             DVEK2170
C                  WILL BE CAUSED WHEN THE NUMBER OF FUNCTION          DVEK2180
C                  EVALUATIONS EXCEEDS ABS(C(7)).                      DVEK2190
C          C(8)  - INTERRUPT NUMBER 1 .  IF NOT ZERO, THE SUB-         DVEK2200
C                  ROUTINE WILL INTERRUPT THE CALCULATIONS             DVEK2210
C                  AFTER IT HAS CHOSEN ITS PRELIMINARY VALUE           DVEK2220
C                  OF HMAG, AND JUST BEFORE CHOOSING HTRIAL            DVEK2230
C                  AND XTRIAL IN PREPARATION FOR TAKING A STEP         DVEK2240
C                  (HTRIAL MAY DIFFER FROM HMAG IN SIGN, AND           DVEK2250
C                  MAY REQUIRE ADJUSTMENT IF XEND IS NEAR).            DVEK2260
C                  THE SUBROUTINE RETURNS WITH IND = 4, AND            DVEK2270
C                  WILL RESUME CALCULATION AT THE POINT OF             DVEK2280
C                  INTERRUPTION IF  RE-ENTERED WITH IND = 4.           DVEK2290
C          C(9)  - INTERRUPT NUMBER 2.  IF NOT ZERO, THE SUB-          DVEK2300
C                  ROUTINE WILL INTERRUPT THE CALCULATIONS             DVEK2310
C                  IMMEDIATELY AFTER IT HAS DECIDED WHETHER OR         DVEK2320
C                  NOT TO ACCEPT THE RESULT OF THE MOST RECENT         DVEK2330
C                  TRIAL STEP, WITH IND = 5 IF IT PLANS TO             DVEK2340
C                  ACCEPT, OR IND = 6 IF IT PLANS TO REJECT.           DVEK2350
C                  Y(*) IS THE PREVIOUSLY ACCEPTED RESULT.             DVEK2360
C                  WHILE W(*,9) IS THE NEWLY COMPUTED TRIAL            DVEK2370
C                  VALUE, AND W(*,2) IS THE UNWEIGHTED ERROR           DVEK2380
C                  ESTIMATE VECTOR.  THE SUBROUTINE WILL RESUME        DVEK2390
C                  CALCULATIONS AT THE POINT OF INTERRUPTION           DVEK2400
C                  ON RE-ENTRY WITH IND = 5 OR 6.                      DVEK2410
C                  IND MAY BE CHANGED BY THE USER IN ORDER TO          DVEK2420
C                  FORCE ACCEPTANCE OF A STEP (BY CHANGING IND         DVEK2430
C                  FROM 6 TO 5) THAT WOULD OTHERWISE BE                DVEK2440
C                  REJECTED, OR VICE VERSA.                            DVEK2450
C          NW    - THE FIRST DIMENSION OF W AS IT APPEARS IN THE       DVEK2460
C                  CALLING PROGRAM.  (INPUT)                           DVEK2470
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C                        NW MUST BE GREATER THAN OR EQUAL TO N.          D/EK2480
C                W       - WORKSPACE MATRIX.                             D/EK2490
C                          THE FIRST DIMENSION OF W MUST BE NW AND THE   D/EK2500
C                          SECOND MUST BE GREATER THAN OR EQUAL TO 9.    D/EK2510
C                IER     - ERROR PARAMETER. (OUTPUT)                     D/EK2520
C                          TERMINAL ERRORS                               D/EK2530
C                            IER = 129. NW IS LESS THAN N OR TOL IS LESS D/EK2540
C                                       THAN OR EQUAL TO ZERO.           D/EK2550
C                            IER = 130. IND IS NOT IN THE RANGE 1 TO 6.  D/EK2560
C                            IER = 131. XEND HAS NOT BEEN CHANGED FROM   D/EK2570
C                                       PREVIOUS CALL OR X IS NOT SET TO D/EK2580
C                                       THE PREVIOUS XEND VALUE.         D/EK2590
C                            IER = 132. THE RELATIVE ERROR CONTROL       D/EK2600
C                                       OPTION (C(1)=2) WAS SELECTED AND D/EK2610
C                                       ONE OF THE SOLUTION COMPONENTS   D/EK2620
C                                       IS ZERO.                         D/EK2630
C   PRECISION            - SINGLE                                         D/EK2640
C   REQD. IMSL ROUTINES  - UERTST                                         D/EK2650
C   LANGUAGE             - FORTRAN                                        D/EK2660
C---------------------------------------------------------------------   D/EK2670
C   LATEST REVISION      - DECEMBER 15, 1976                              D/EK2680
C                               BGH                                      D/EK2690
      INTEGER              N,IND,NW,K                                     D/EK2700
      INTEGER              IER                                            D/EK2710
      REAL                 X,Y(N),XEND,TOL,C(1),W(NW,9),TEMP              D/EK2720
      REAL                 ZERO,ONE,TWO,THREE,FOUR,FIVE,SEVEN,TEN,HALF,P9.D/EK2730
     1                     C4D15,C2D3,C5D6,C1D6,C1D15,C2D96               D/EK2740
      REAL                 RK(39),REPS,RTOL                               D/EK2750
      DATA                 ZERO/0.0/,ONE/1.0/,TWO/2.0/,THREE/3.0/         D/EK2760
      DATA                 FOUR/4.0/,FIVE/5.0/,SEVEN/7.0/                 D/EK2770
      DATA                 TEN/10.0/,HALF/0.5/,P9/0.9/                    D/EK2780
      DATA                 C4D15/.26666666667/                            D/EK2790
      DATA                 C2D3/.66666666667/                             D/EK2800
      DATA                 C5D6/.83333333333/                             D/EK2810
      DATA                 C1D6/.16666666667/                             D/EK2820
      DATA                 C1D15/.66666666667E-1/                         D/EK2830
      DATA                 C2D96/120.42729108/                            D/EK2840
      DATA                 REPS/013010000000000000/                       D/EK2850
      DATA                 RTOL/016310000000000000/                       D/EK2860
      DATA                 RK( 1)/.16666666667E+00/                       D/EK2870
      DATA                 RK( 2)/.53333333333E-01/                       D/EK2880
      DATA                 RK( 3)/.21333333333E+00/                       D/EK2890
      DATA                 RK( 4)/.83333333333E+00/                       D/EK2900
      DATA                 RK( 5)/.26666666667E+01/                       D/EK2910
      DATA                 RK( 6)/.25000000000E+01/                       D/EK2920
      DATA                 RK( 7)/.25781250000E+01/                       D/EK2930
      DATA                 RK( 8)/.91666666667E+01/                       D/EK2940
      DATA                 RK( 9)/.66406250000E+01/                       D/EK2950
      DATA                 RK(10)/.88541666667E+00/                       D/EK2960
      DATA                 RK(11)/.24000000000E+01/                       D/EK2970
      DATA                 RK(12)/.80000000000E+01/                       D/EK2980
      DATA                 RK(13)/.65604575163E+01/                       D/EK2990
      DATA                 RK(14)/.30555555556E+00/                       D/EK3000
      DATA                 RK(15)/.34509803922E+00/                       D/EK3010
      DATA                 RK(16)/.55086666667E+00/                       D/EK3020
      DATA                 RK(17)/.16533333333E+01/                       D/EK3030
      DATA                 RK(18)/.94558823529E+00/                       D/EK3040
      DATA                 RK(19)/.32400000000E+00/                       D/EK3050
      DATA                 RK(20)/.23378823529E+00/                       D/EK3060
      DATA                 RK(21)/.20354651163E+01/                       D/EK3070
      DATA                 RK(22)/.69767441860E+01/                       D/EK3080
      DATA                 RK(23)/.56481798146E+01/                       D/EK3090
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      DATA                RK(24)/.13738156761E+00/                          DVEK3100
      DATA                RK(25)/.28630226610E+00/                          DVEK3110
      DATA                RK(26)/.14417855672E+00/                          DVEK3120
      DATA                RK(27)/.75000000000E-01/                          DVEK3130
      DATA                RK(28)/.38992869875E+00/                          DVEK3140
      DATA                RK(29)/.31944444444E+00/                          DVEK3150
      DATA                RK(30)/.13503836317E+00/                          DVEK3160
      DATA                RK(31)/.10783298827E-01/                          DVEK3170
      DATA                RK(32)/.69805194805E-01/                          DVEK3180
      DATA                RK(33)/.62500000000E-02/                          DVEK3190
      DATA                RK(34)/.69630124777E-02/                          DVEK3200
      DATA                RK(35)/.69444144444E-02/                          DVEK3210
      DATA                RK(36)/.61381074169E-02/                          DVEK3220
      DATA                RK(37)/.68181818182E-01/                          DVEK3230
      DATA                RK(38)/.10783298827E-01/                          DVEK3240
      DATA                RK(39)/.69805194805E-01/                          DVEK3250
C                                                                           DVEK3260
C                                 BEGIN INITIALIZATION, PARAMETER           DVEK3270
C                                   CHECKING, INTERRUPT RE-ENTRIES          DVEK3280
      IER = 0                                                               DVEK3290
C                                 ABORT IF IND OUT OF RANGE 1 TO 6          DVEK3300
      IF (IND.LT.1.OR.IND.GT.6) GO TO 290                                   DVEK3310
C                                 CASES - INITIAL ENTRY, NORMAL             DVEK3320
C                                   RE-ENTRY, INTERRUPT RE-ENTRIES          DVEK3330
      GO TO (5,5,40,145,265,265), IND                                       DVEK3340
C                                 CASE 1 - INITIAL ENTRY (IND .EQ. 1        DVEK3350
C                                   OR 2) ABORT IF N.GT.NW OR TOL.LE.0      DVEK3360
C                                                                           DVEK3370
    5 IF (N.GT.NW.OR.TOL.LE.ZERO) GO TO 295                                 DVEK3380
      IF (IND.EQ.2) GO TO 15                                                DVEK3390
C                                 INITIAL ENTRY WITHOUT OPTIONS (IND        DVEK3400
C                                   .EQ. 1) SET C(1) TO C(9) EQUAL TO       DVEK3410
C                                   0                                       DVEK3420
      DO 10 K=1,9                                                           DVEK3430
         C(K) = ZERO                                                        DVEK3440
   10 CONTINUE                                                              DVEK3450
      GO TO 30                                                              DVEK3460
C                                 SUMMARY OF THE COMPONENTS OF THE          DVEK3470
C                                   COMMUNICATIONS VECTOR                   DVEK3480
C                                   PRESCRIBED AT THE OPTION                DVEK3490
C                                     OF THE USER                           DVEK3500
C                                                                           DVEK3510
C                                   C(1) ERROR CONTROL INDICATOR            DVEK3520
C                                   C(2) FLOOR VALUE                        DVEK3530
C                                   C(3) HMIN SPECIFICATION                 DVEK3540
C                                   C(4) HSTART SPECIFICATION               DVEK3550
C                                   C(5) SCALE SPECIFICATION                DVEK3560
C                                   C(6) HMAX SPECIFICATION                 DVEK3570
C                                   C(7) MAX NO OF FCN EVALS                DVEK3580
C                                   C(8) INTERRUPT NO 1                     DVEK3590
C                                   C(9) INTERRUPT NO 2                     DVEK3600
C                                                                           DVEK3610
C                                   DETERMINED BY THE PROGRAM               DVEK3620
C                                                                           DVEK3630
C                                   C(10) RREB(REL ROUNDOFF ERROR BND)      DVEK3640
C                                   C(11) DWARF (VERY SMALL MACH NO)        DVEK3650
C                                   C(12) WEIGHTED NORM Y                   DVEK3660
C                                   C(13) HMIN                              DVEK3670
C                                   C(14) HMAG                              DVEK3680
C                                   C(15) SCALE                             DVEK3690
C                                   C(16) HMAX                              DVEK3700
C                                   C(17) XTRIAL                            DVEK3710
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C                                C(18) HTRIAL                          D/EK3720
C                                C(19) EST                             D/EK3730
C                                C(20) PREVIOUS XEND                   D/EK3740
C                                C(21) FLAG FOR XEND                   D/EK3750
C                                C(22) NO OF SUCCESSFUL STEPS          D/EK3760
C                                C(23) NO OF SUCCESSIVE FAILURES       D/EK3770
C                                C(24) NO OF FCN EVALS                 D/EK3780
C                                IF C(1) = 4 OR 5, C(31),C(32)....     D/EK3790
C                                  C(N+30) ARE FLOOR VALUES            D/EK3800
   15 CONTINUE                                                         D/EK3810
C                              INITIAL ENTRY WITH OPTIONS (IND .EQ.    D/EK3820
```

Appendix D

Material Property Data

It was necessary to perform a series of elastic modulus determination tests to characterize this adherent material. Slight variations in material properties can be evident in molding compounds even manufactured by the same supplier. In a separate, extensive study concerning material property data, Taggart reported the elastic modulus of SMC-25 to be 2.1×10^6 psi and the results shown in Table 4 are in close agreement.

Table 4

Specimen #	Modulus (PSI)
SPEC1	2.21×10^6
SPEC2	2.26×10^6
SPEC3	2.18×10^6

Plate 6 shows a typical test specimen used for modulus determination. The data for these tests may be found on the following pages.

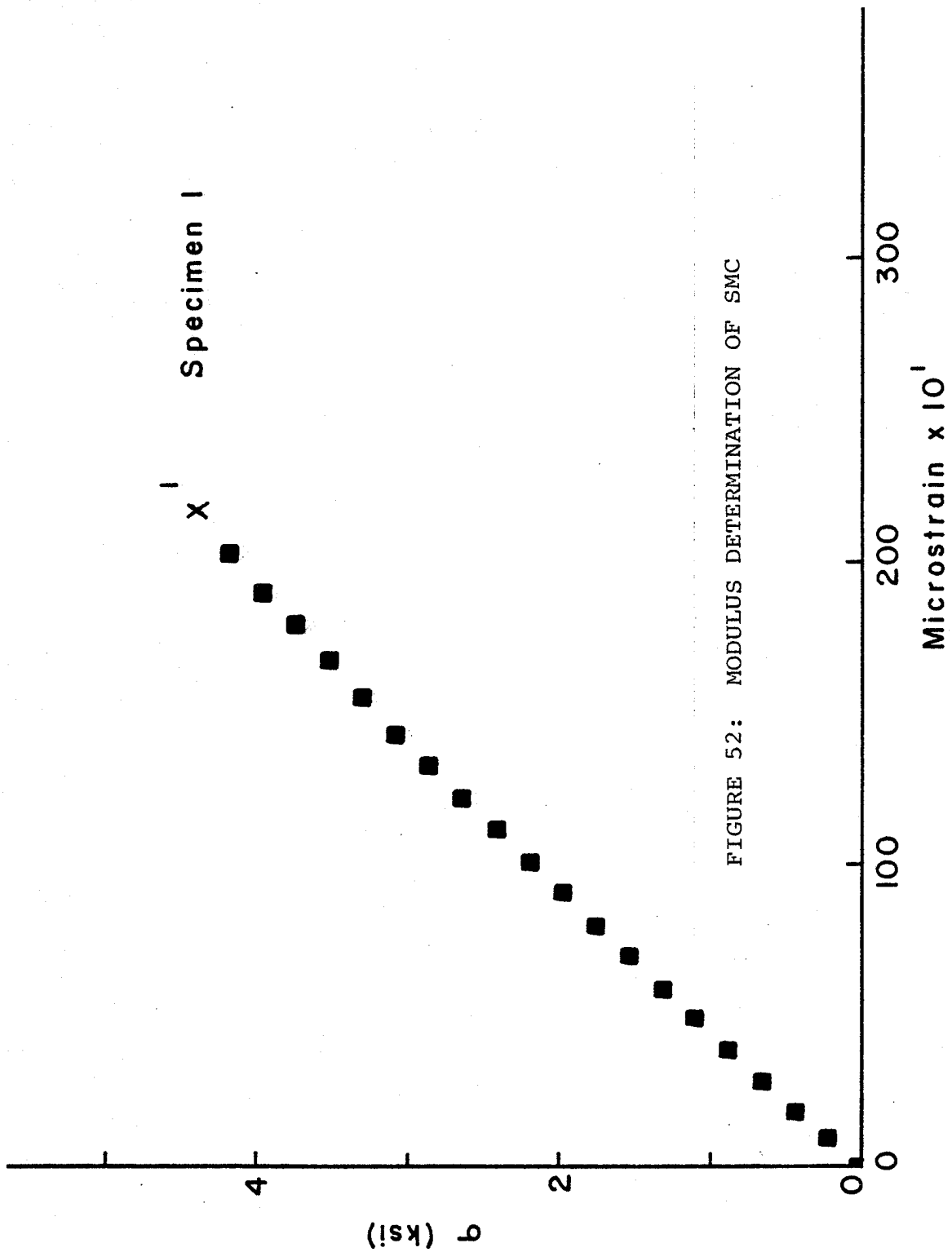


FIGURE 52: MODULUS DETERMINATION OF SMC

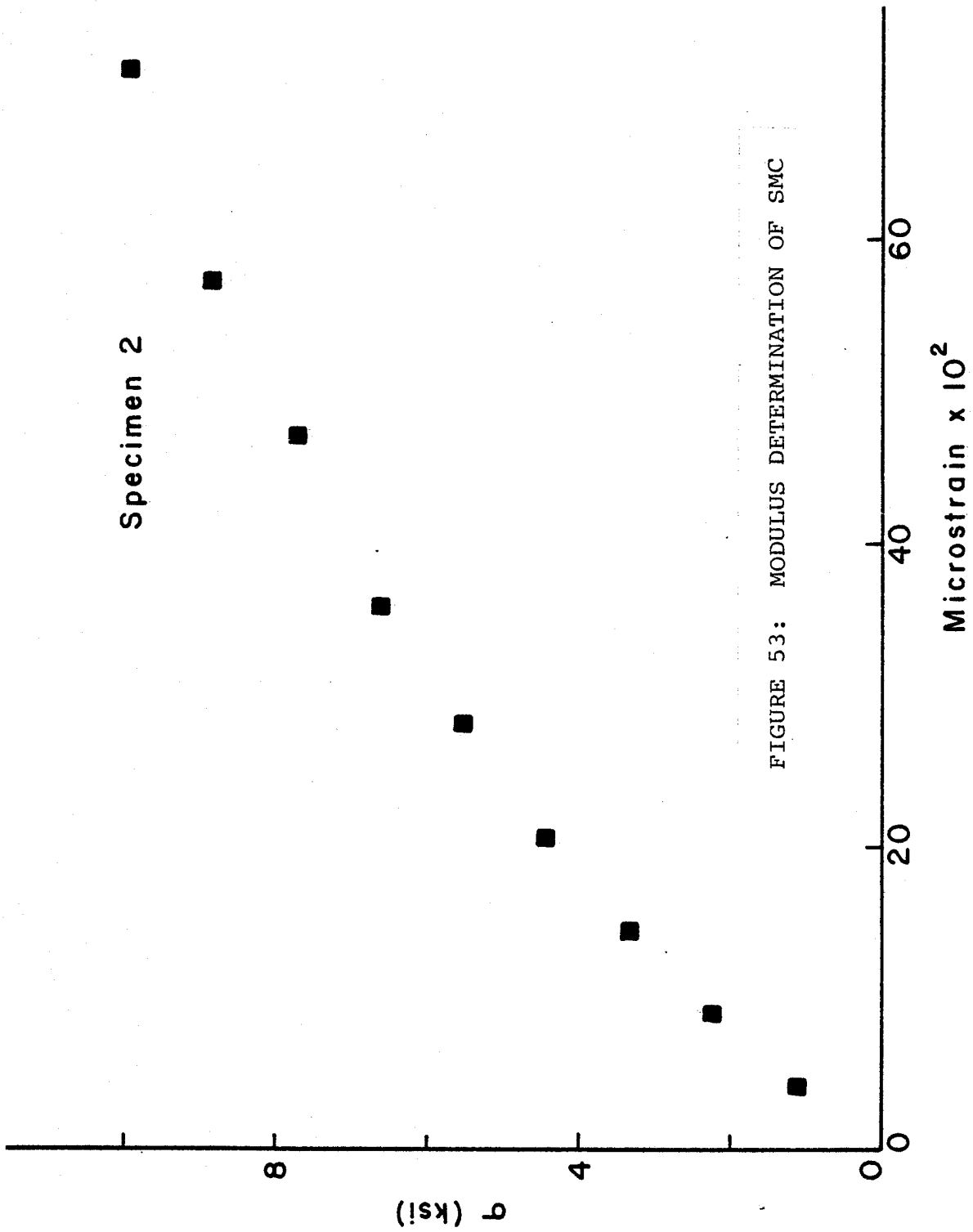


FIGURE 53: MODULUS DETERMINATION OF SMC

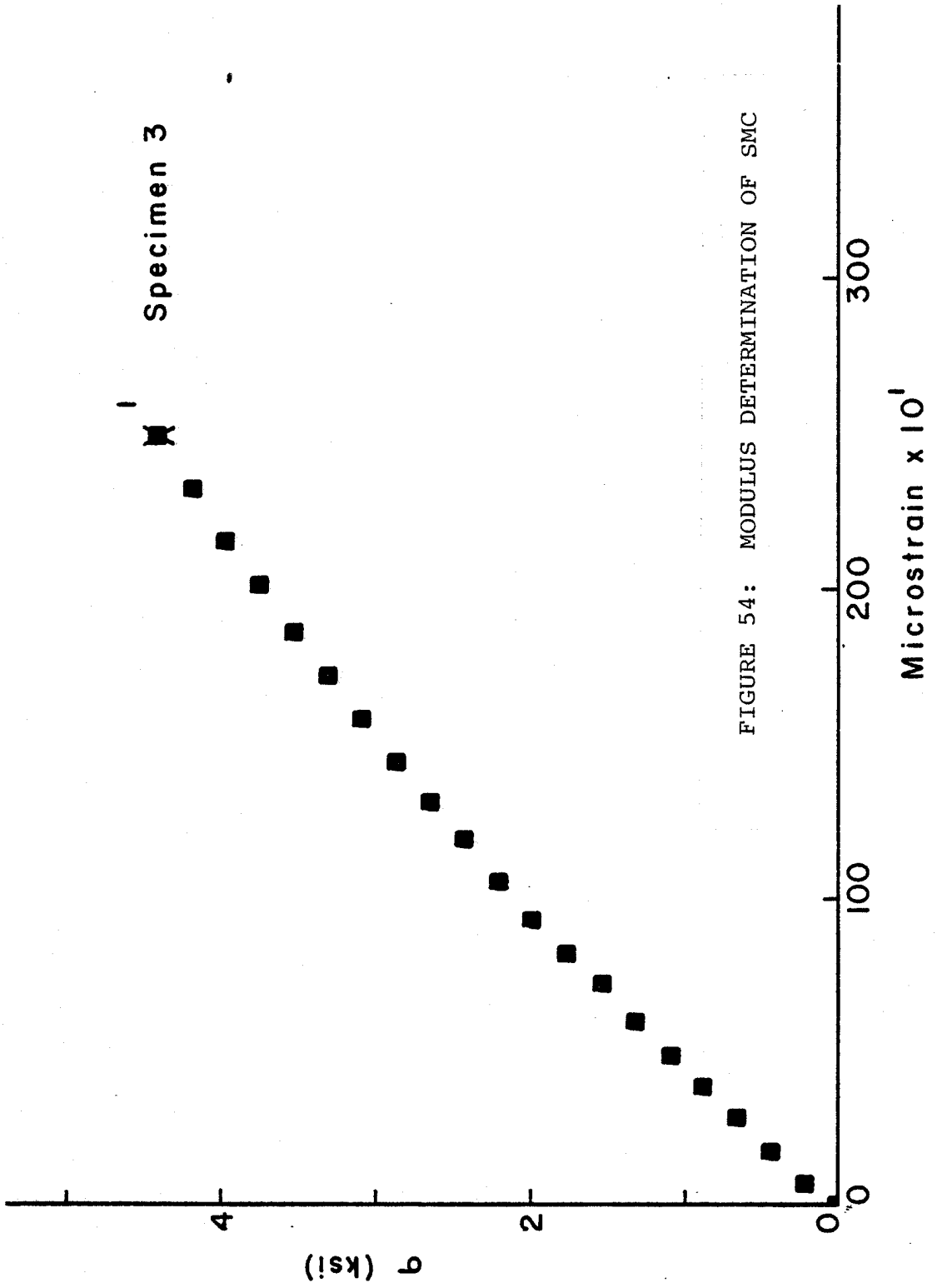


FIGURE 54: MODULUS DETERMINATION OF SMC

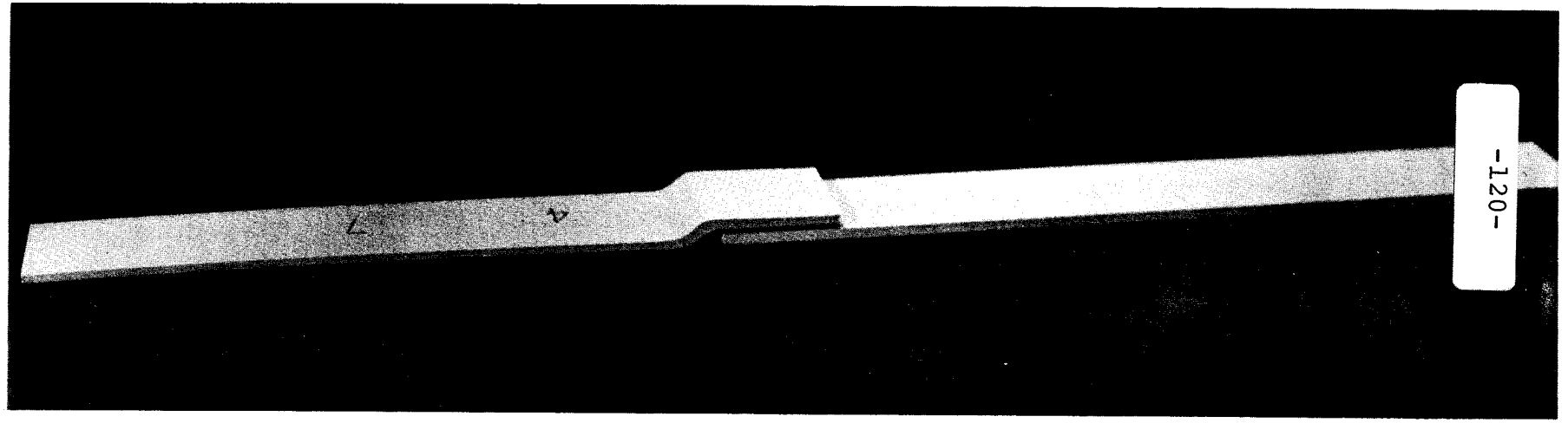


PLATE 1: Typical Test Specimen

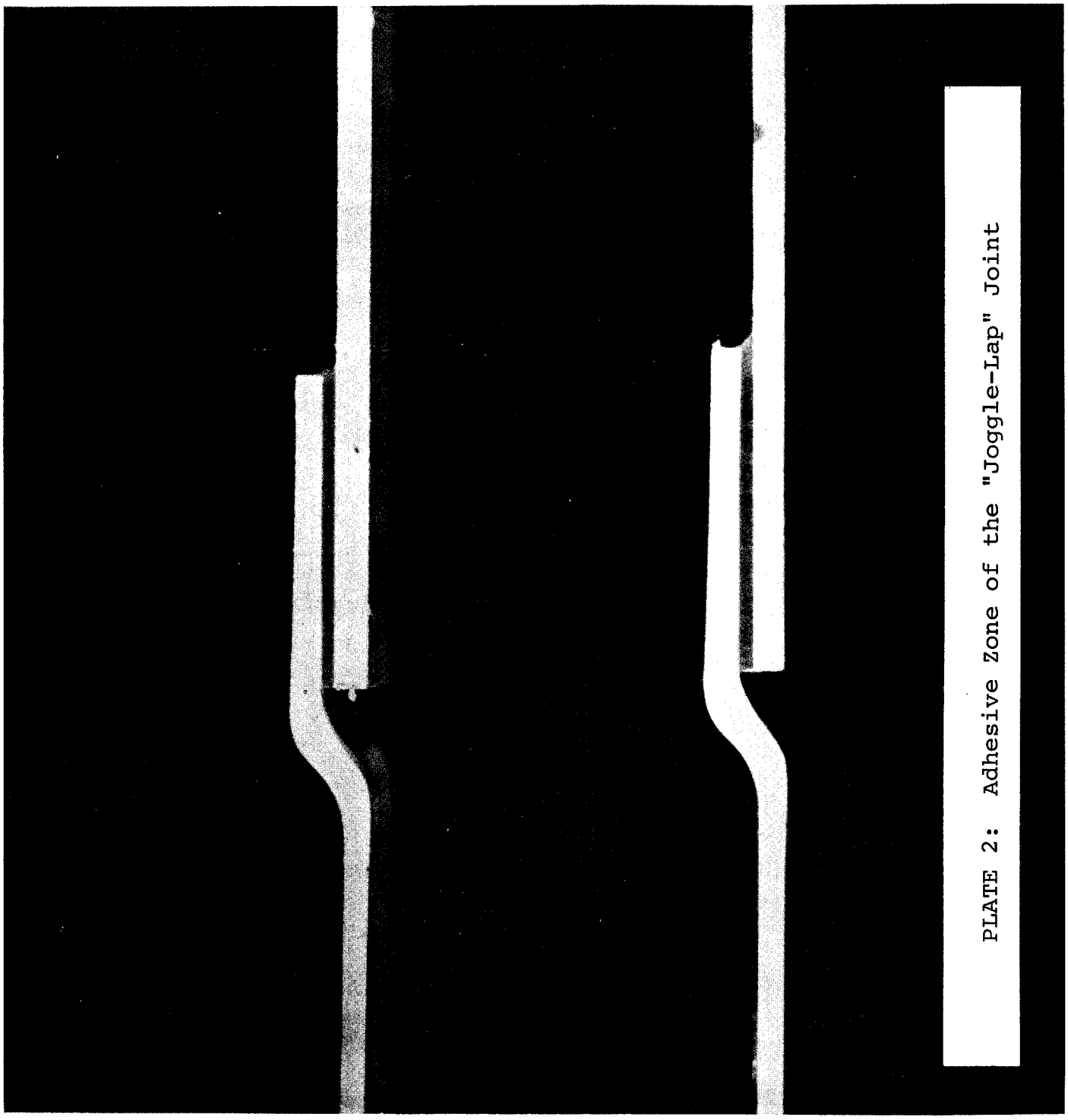


PLATE 2: Adhesive Zone of the "Joggle-Lap" Joint

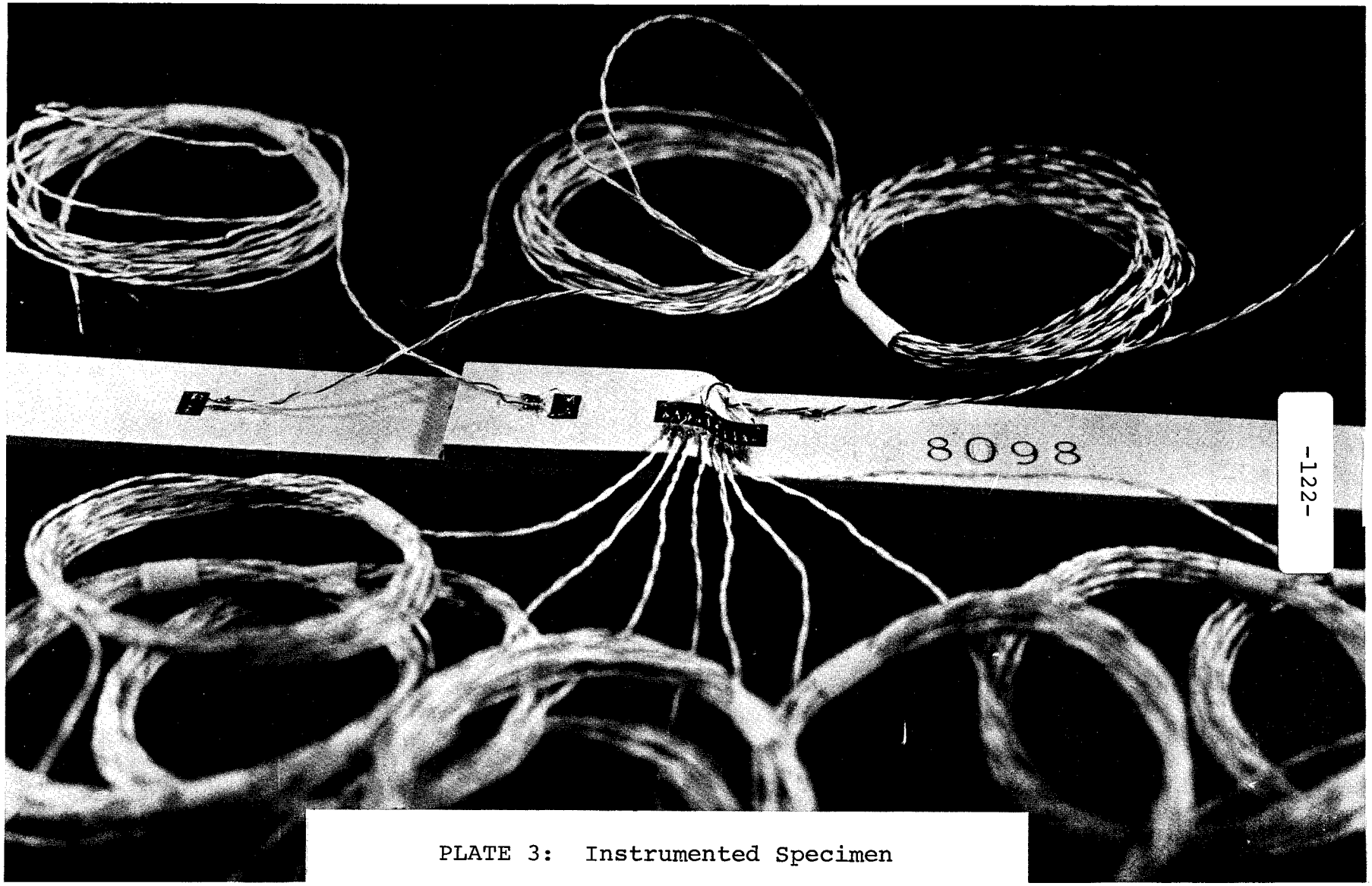


PLATE 3: Instrumented Specimen

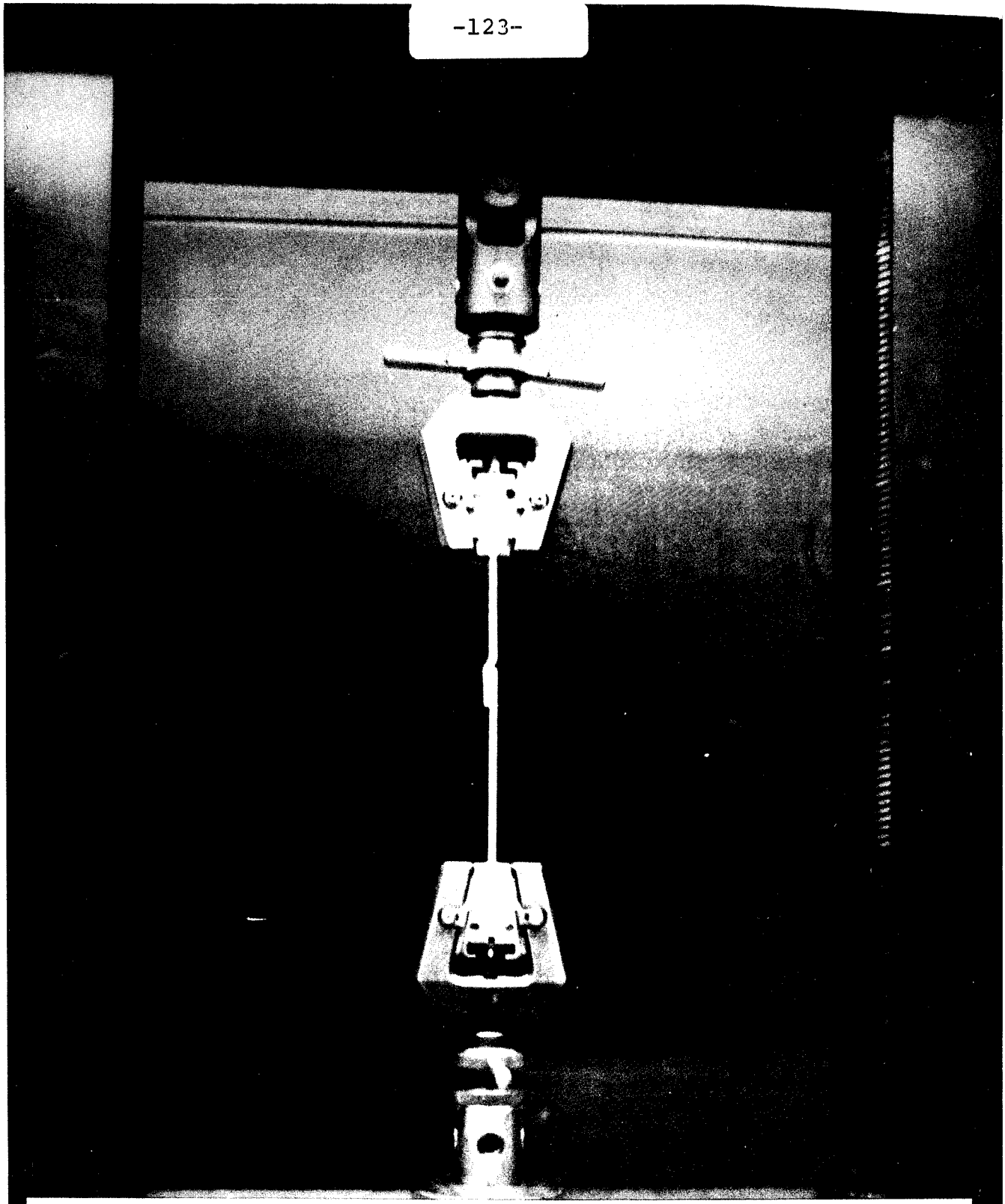


PLATE 4: "Joggle-Lap" Joint Subject to Tension

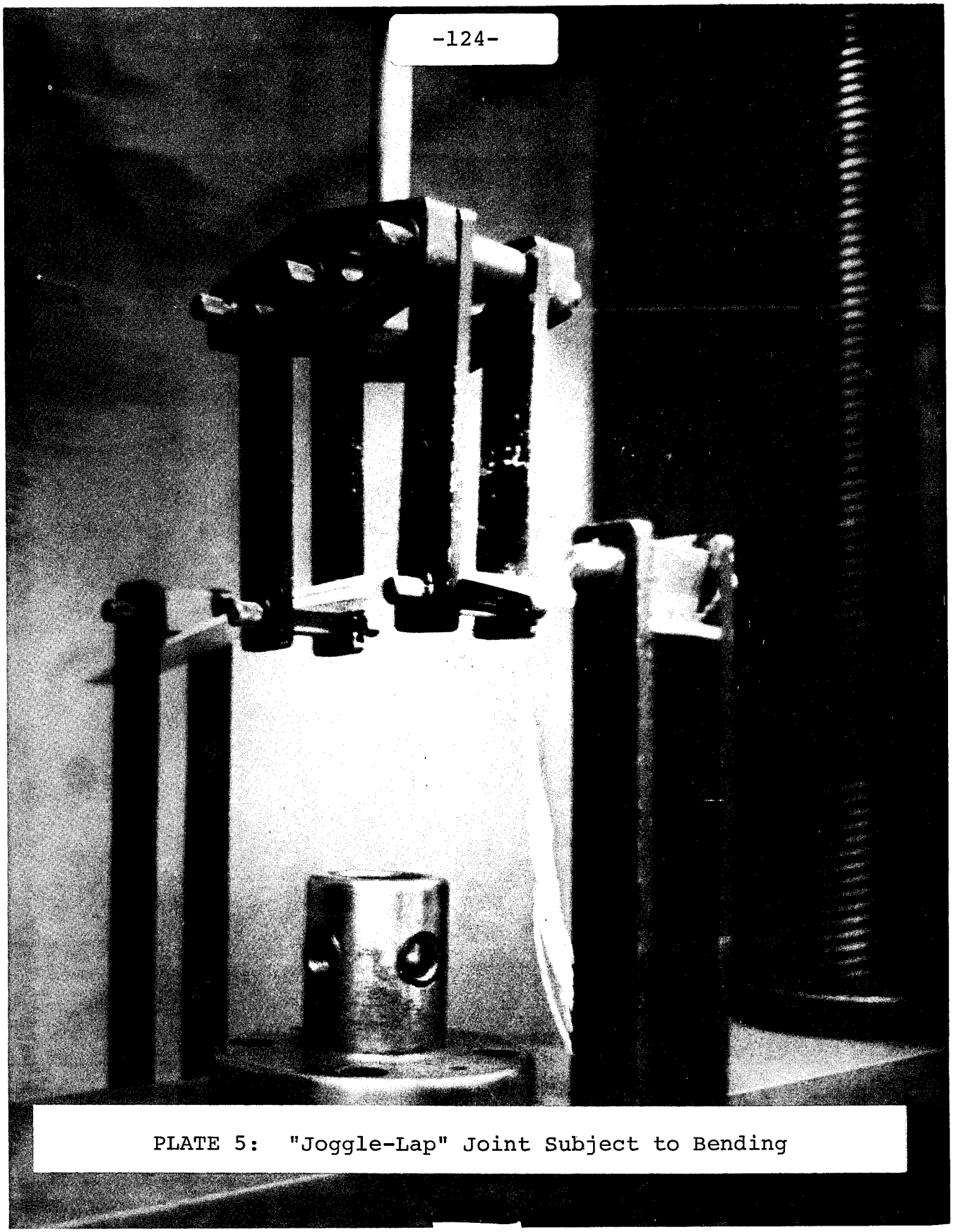


PLATE 5: "Joggle-Lap" Joint Subject to Bending

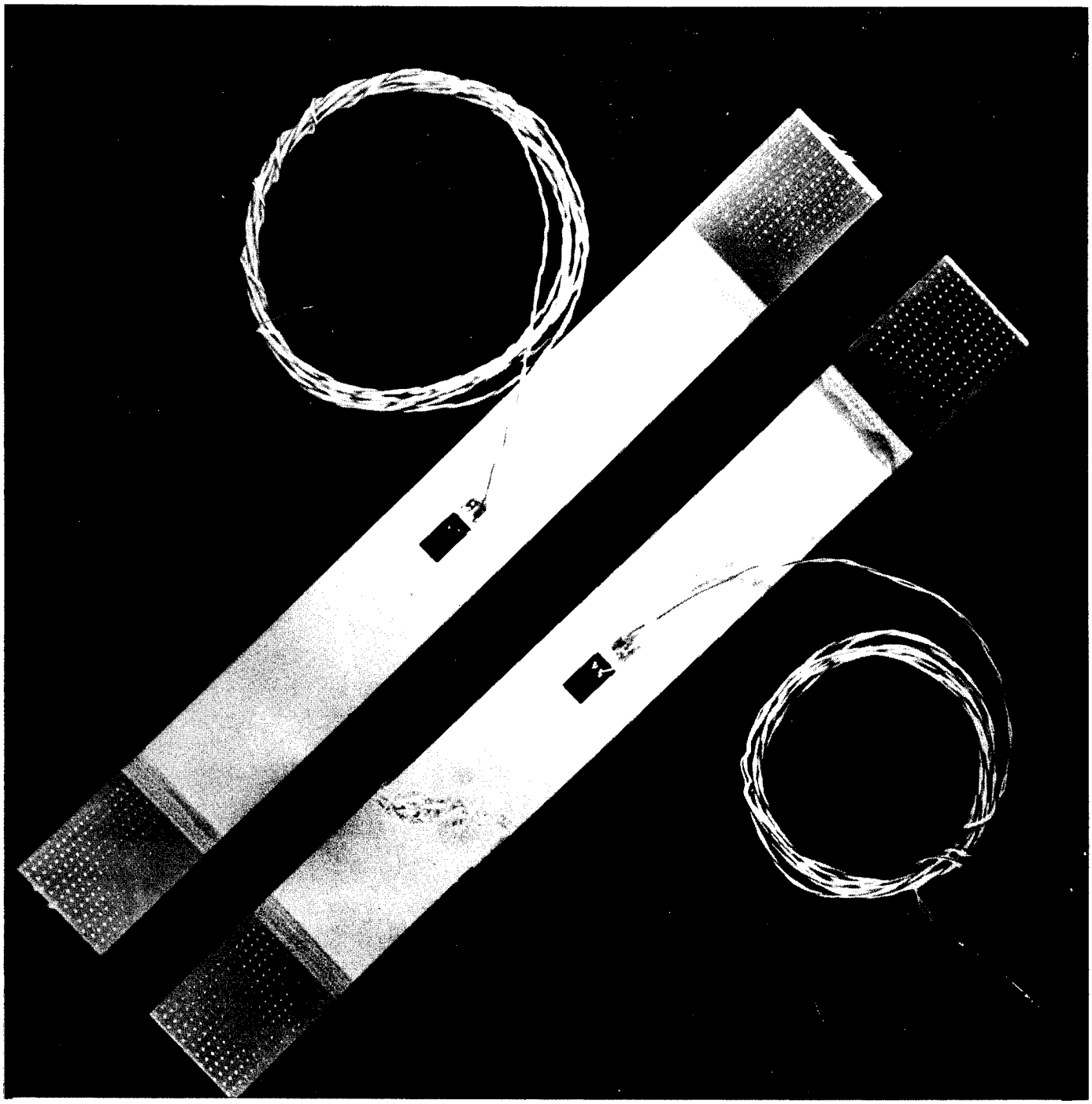
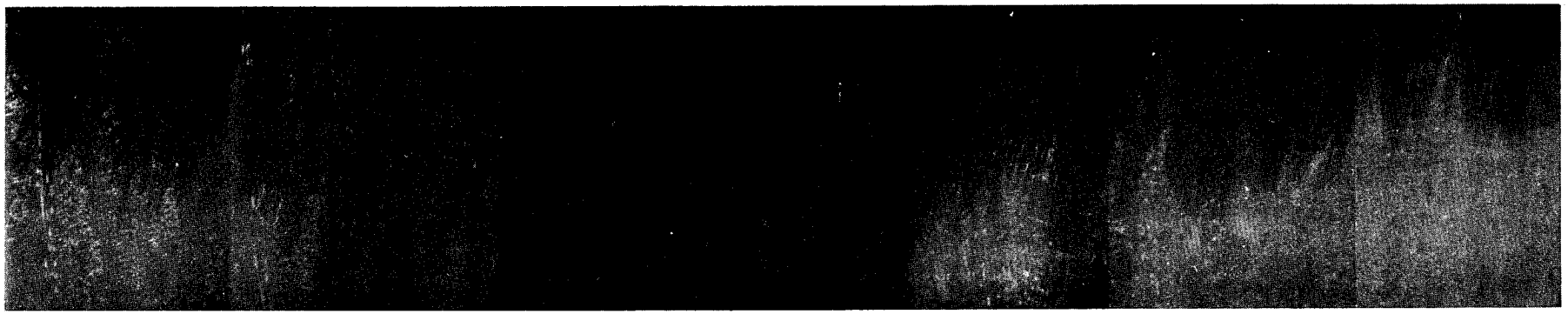


PLATE 6: Tensile Coupons for Modulus Determination



Curved Section (SEG3)

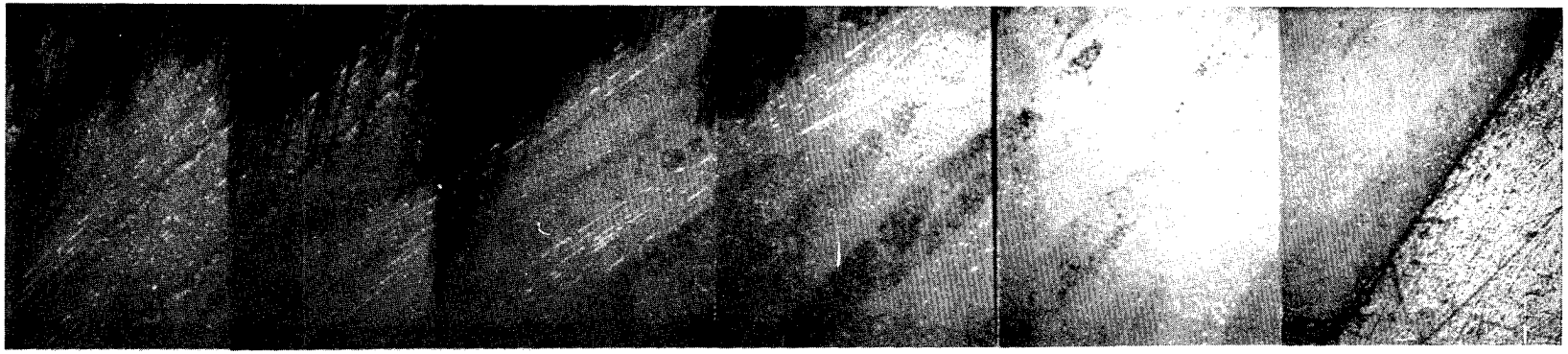


PLATE 7: Photomicrographs Showing Relative Fiber Content